7. DELAY AND LEVEL OF SERVICE

Learning objectives:
- Calculate uniform delay at a signalized intersection
- Describe and apply measures of effectiveness for a signalized intersection
- Describe and apply the level of service framework

Just as we earlier asked the question "is there sufficient capacity at this intersection to accommodate demand", we can also ask another related question: "how is the intersection performing?" Or, stated another way: "what quality of service is provided to the users of the intersection?" Sometimes this question is asked for one intersection and its individual movements. Often it is asked when we are comparing alternative designs for an intersection or comparing the operation of several different intersections. Three performance measures are commonly used to evaluate intersection performance: delay, volume-to-capacity ratio, and back of queue size. Delay is a measure that can be perceived by users; it is used to determine level of service.

7.1 Determining Uniform Delay Using the Cumulative Vehicle Diagram and the Queue Accumulation Polygon

In section 2, the cumulative vehicle diagram and the queue accumulation polygon were presented as two ways of representing traffic flow at a signalized intersection. Figure 1 shows a cumulative vehicle diagram highlighting the cumulative number of vehicles that have arrived at and departed from the signalized intersection over time. The slope of the cumulative arrival line (solid line) is equal to the arrival rate v. Three time periods are shown for the cumulative departure line:

- Effective red (period 1), during which the departure flow is zero,
- The queue service time g_s (period 2), in which the slope of the cumulative departure line is equal to the saturation flow rate s, and
- The final portion of effective green (period 3), in which the slopes of the cumulative arrival line and the cumulative departure line are equal to the arrival rate v.

The horizontal line connecting the arrival line and the departure line for each vehicle (shown in Figure 1 as d_i) is the delay experienced by that vehicle. The length of the queue, measured in vehicles, is the vertical distance at a given point in time between the arrival line and the departure line. The area of the triangle is equal to the total delay experienced by all vehicles that arrive during the cycle. The delay is called uniform delay since vehicles are assumed to arrive at the intersection at a uniform flow rate.

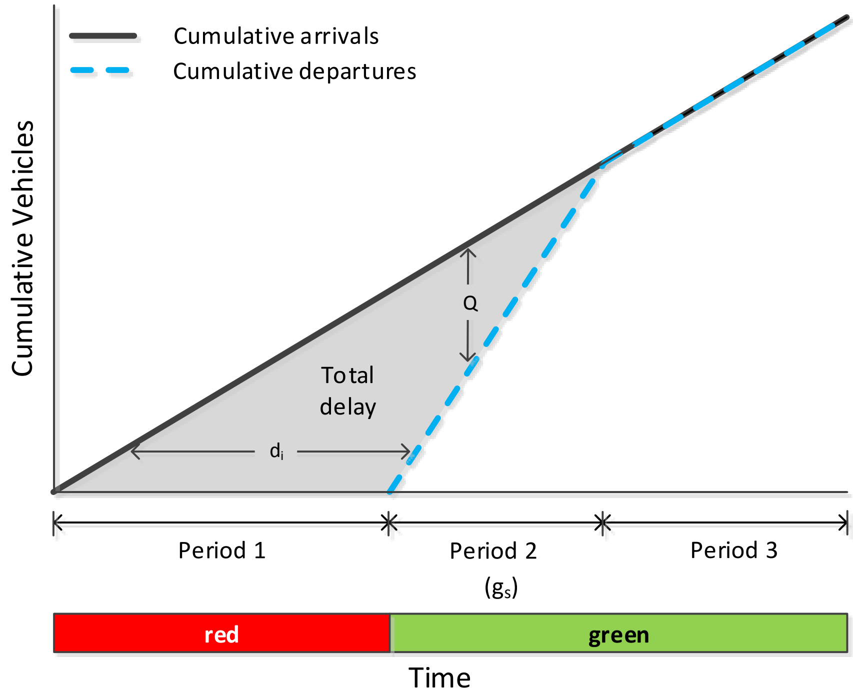

Figure 1. Cumulative vehicle diagram

Figure 2 shows a queue accumulation polygon for the same traffic flow conditions and time periods as shown in Figure 1. Here, the height of the polygon is the length of the queue at any point in time. The maximum queue length occurs at the end of the effective red interval. The area of the triangle is equal to the total delay experienced by all vehicles that arrive during the cycle.

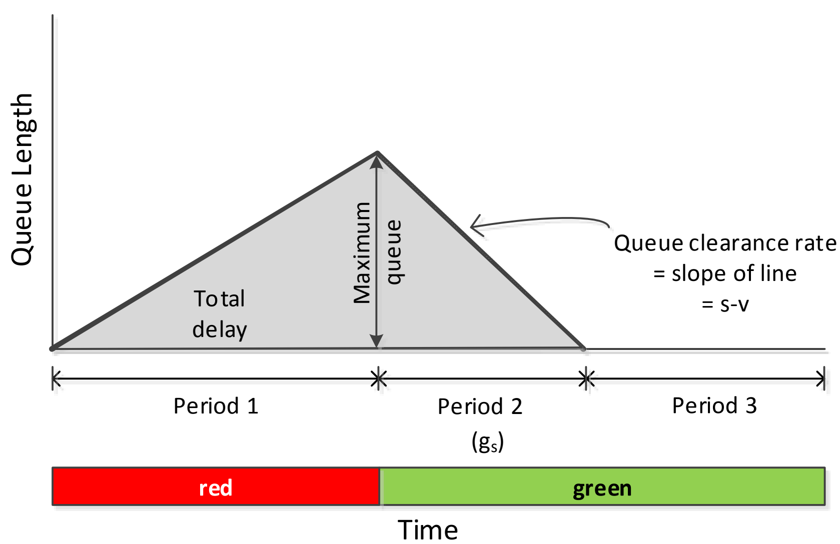

Figure 2. Queue accumulation polygon

Two assumptions made about the D/D/1 queuing model are represented in these diagrams:

- The queue clears before the end of the effective green, implying that the arrival volume is less than the capacity. This also implies that the queue at the beginning of red is zero.
- The arrival pattern is uniform.

The first step in determining the area of the triangle (total delay) is to compute the time that it takes for the queue to clear after the beginning of effective green. We call this time the queue service time g_s. When the queue clears, the number of vehicles that have arrived at the intersection since the beginning of the cycle must equal the number of vehicles that have departed since the beginning of the effective green. We can write this equality as

Equation 1

$$v(r + g_s) = sg_s$$

where

v = arrival rate, veh/sec,
r = effective red time, sec,
g_s = queue service time, sec, and
s = saturation flow rate, veh/sec.

Solving for g_s:

Equation 2

$$g_s = \frac{vr}{s - v}$$

Stated in words, the queue service time is equal to the length of the queue at the end of effective red (*vr)* divided by the rate of queue clearance after the start of effective green (*s*-*v*).

The area of the triangle in Figure 1 is equal to one-half the product of the effective red time (the base of the triangle) and the number of vehicles that have arrived at the intersection at the point that the queue has cleared, $v(r + g_s)$. This latter number is the height of the triangle. The total uniform delay D_t is given in Equation 3.

Equation 3

$$D_t = (0.5)(r)[(v)(r + g_s)]$$

where the variables are defined as above. Substituting g_s from Equation 2, we get another expression for the total delay.

Equation 4

$$D_t = (0.5)(r)\left[(v)\left(r + \frac{vr}{s - v}\right)\right]$$

The number of vehicles that arrive during the cycle is the product of the arrival rate *v* and the cycle length *C*. The *average uniform delay per vehicle* d_{avg} is the total delay from Equation 4 divided by the number of vehicles that arrive during the cycle (*vC)*.

Equation 5

$$d_{avg} = \frac{(0.5)(r)}{vC}\left[(v)\left(r + \frac{vr}{s - v}\right)\right]$$

When terms are rearranged and simplified:

Equation 6

$$d_{avg} = (0.5r)\left[\frac{(1 - g/C)}{(1 - v/s)}\right]$$

The same equation will result if we compute the area from the queue accumulation polygon in Figure 2. These equations yield the average delay experienced by vehicles if the arrival pattern is uniform and if the demand is less than the capacity of the approach. When we examine the equation for average delay, we can see that delay increases when:

- The effective red increases, resulting in a longer queue that forms during red.
- The effective green ratio decreases, providing less green time during the cycle to serve the queue.
- The flow ratio increases, as the volume approaches the capacity.

The average uniform delay for the entire intersection can also be computed. If the delay for each approach is computed to be d_i and the volume on each approach is v_i, the average delay for the intersection d_{int} is the weighted average of the delays for each of the intersection approaches.

Equation 7

$$d_{int} = \frac{\sum d_i v_i}{\sum v_i}$$

where

d_i = delay for each of the i approaches at the intersection, sec, and

v_i = the volume for each of these approaches, veh/sec.

Example 13. Calculation Of Average Delay When Volume Is Less Than Capacity

An intersection approach has an arrival rate of 630 veh/hr and a saturation flow rate of 1900 veh/hr. The cycle length is 100 sec, the effective red time is 60 sec, and the effective green time is 40 sec. Determine the queue service time and the average delay for this approach.

Step 1. Convert the flow rates from veh/hr to veh/sec.

$$v = \frac{630\ veh/hr}{3600\ sec/hr} = .175\ veh/sec$$

$$s = \frac{1900\ veh/hr}{3600\ sec/hr} = .528\ veh/sec$$

Step 2. Using **Error! Reference source not found.**, calculate the approach capacity and compare it to the arrival flow rate.

$$c = s \times \left(\frac{g}{C}\right) = 1900\ veh/hr\ \times \left(\frac{40\ sec}{100\ sec}\right) = 760\ veh/hr$$

The arrival volume (630 veh/hr) is thus less than the capacity (760 veh/hr), so the analytical method (Equation 6) can be used to calculate the average uniform delay.

Step 3. Calculate the queue service time using Equation 2.

$$g_s = \frac{vr}{s - v} = \frac{(.175\ veh/sec)(60\ \mathrm{sec})}{.528\ veh/sec - .175\ veh/sec} = 29.7\ sec$$

Step 4. Construct the cumulative vehicle diagram and the queue accumulation polygon. (Though this step is not necessary to solve this problem, the preparation of the two diagrams is recommended for better understanding these concepts).

Cumulative vehicle diagram: The arrival line (solid line, Figure 3) shows the cumulative number of vehicles that arrive from the beginning of the cycle (t=0) to the end of the cycle (t=100). The departure line (dashed line) shows the cumulative number of departures over time. During the effective red (from t=0 to t=60), the number of departures is zero. At the end of the effective red, the queue is 10.5 vehicles (line 1-2). From the beginning of effective green to the time that the queue clears (point 3), the departure line has a slope equal to the saturation flow rate. After the queue has cleared (t=89.7, 29.7 sec after the beginning of effective green), the arrival line and the departure line are coincident.

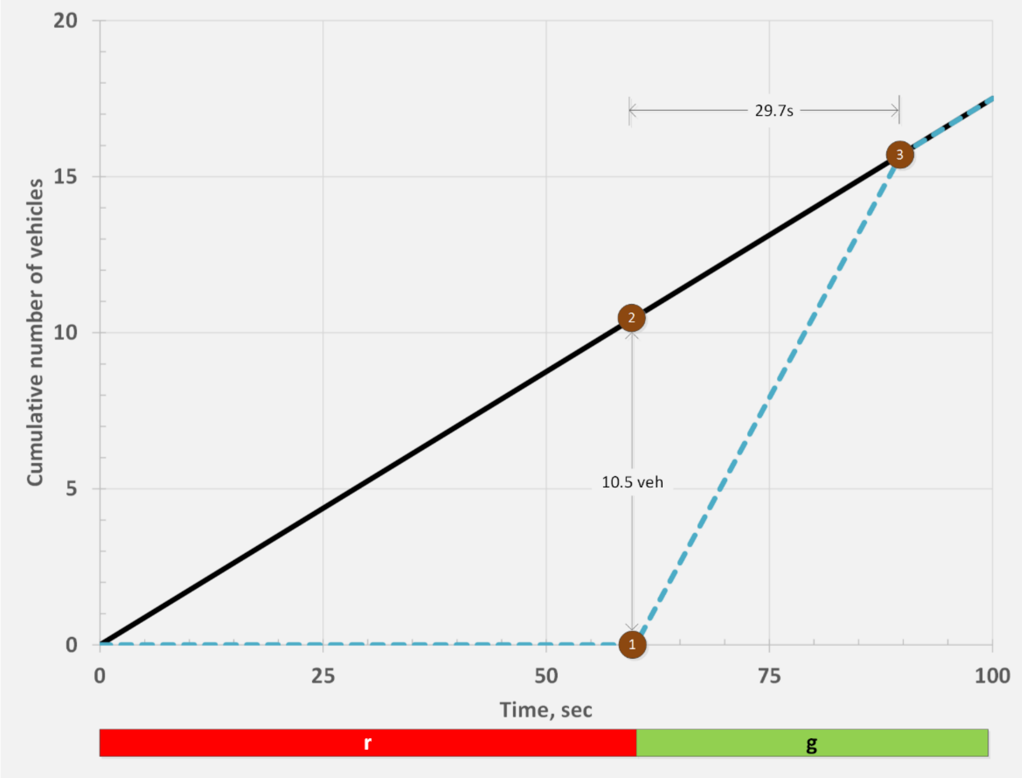

Figure 3. Cumulative vehicle diagram for Example 13

Queue accumulation polygon: The height of the polygon in Figure 4 shows the number of vehicles in the queue at any point during the cycle.

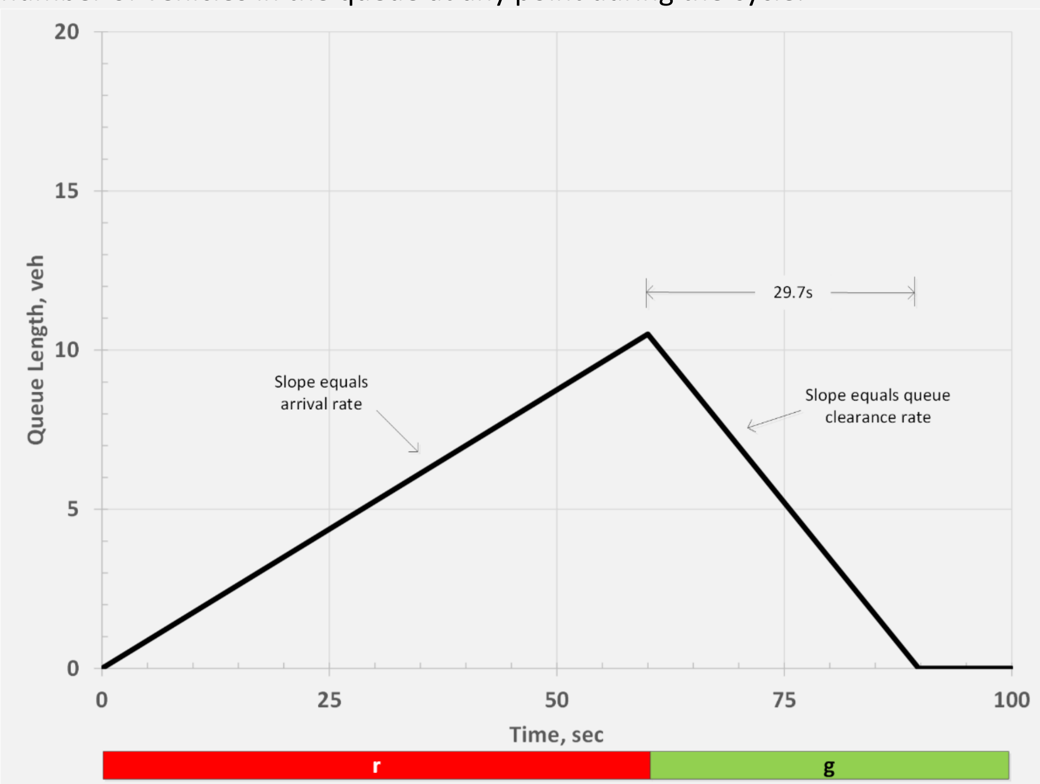

Figure 4. Queue accumulation polygon for Example 13

Step 5. The average delay is calculated using Equation 6:

$$d_{avg} = (0.5r)\left[\frac{\left(1-\frac{g}{C}\right)}{\left(1-\frac{v}{s}\right)}\right]$$

$$d_{avg} = (0.5 \times 60\ sec)\left[\frac{\left(1-\frac{40\ sec}{100\ sec}\right)}{\left(1-\frac{630\ veh/hr}{1900\ veh/hr}\right)}\right] = 26.9\ sec$$

When the volume exceeds the capacity, a basic assumption of the D/D/1 queuing model is violated and we must use the graphical method to compute the delay. The graphical method is based on finding the area of the triangle in either the cumulative vehicle diagram or the queue accumulation polygon. An example of the graphical method is given in Example 14.

Example 14. Calculation Of Average Delay When Volume Exceeds Capacity
An intersection approach has a cycle length of 100 sec and an effective green time of 40 sec. The arrival rate varies over three cycles. The arrival rate is 900 veh/hr during the first cycle, 720 veh/hr during the second cycle, and 540 veh/hr during the third cycle. Calculate the average delay for the approach over the three cycles. The saturation flow rate is 1900 veh/hr.

Step 1. Convert the flow rates from veh/hr to veh/sec. The resulting rates for each cycle are shown in Table 1.

Table 1. Vehicle arrival rates during each cycle

Cycle	Flow rate (veh/hr)	Flow rate (veh/sec)
1	900	.25
2	720	.20
3	540	.15

Step 2. Using **Error! Reference source not found.**, calculate the approach capacity and compare it to the arrival flow rate for each cycle.

$$c = s \times \left(\frac{g}{C}\right) = 1900\ veh/hr\ \times \left(\frac{40\ sec}{100\ sec}\right) = 760\ veh/hr$$

The arrival flow rate is less than the capacity for the second and third cycle. But in the first cycle, the volume of 900 veh/hr exceeds the capacity of 760 veh/hr. Because volume exceeds capacity, Equation 6 cannot be applied to determine the delay. We must instead use the graphical approach of the cumulative vehicle diagram or queue accumulation diagram.

Step 3. Calculate the queue service time g_s for the first cycle using Equation 2.

$$g_s = \frac{vr}{s - v} = \frac{(.250\ veh/sec)(60\ \text{sec})}{.528\ veh/sec - .250\ veh/sec} = 54.0\ sec$$

This result confirms the finding in step 2 that the volume exceeds capacity. The queue would take 54.0 sec to clear, longer than the effective green time of 40 sec.

Step 4. Construct the cumulative vehicle diagram and queue accumulation polygon for the three cycles using the flow rate data given above.

Cumulative vehicle diagram: The arrival line (solid line, Figure 5) shows the cumulative number of vehicles that arrive from the beginning of the first cycle (t=0) to the end of the third cycle (t=300). The departure line (dashed line) shows the cumulative number of departures over time. The number of vehicles in queue at the end of each of the effective red and effective green periods are shown by the vertical lines.

Queue accumulation polygon. Figure 6 shows the variation in the queue length during the three cycles. The queue does not clear by the end of the first cycle, consistent with the calculations shown in steps 2 and 3. The residual queue at this point is 3.9 vehicles. Although the volume is less than the capacity in the second cycle, it is not sufficiently less to allow the queue to clear. There is still a residual queue of 2.8 vehicles at the end of cycle 2.

During the third cycle, the queue finally clears. But since there is a residual queue at the end of the second cycle, the numerator of Equation 2 (used to calculate the queue service time) must include both this residual queue and the queue that forms during effective red in the third cycle. The queue clears 31.2 seconds after the beginning of the effective green.

$$g_s = \frac{(residual\ queue) + vr}{s - v} = \frac{2.8\ veh + (.15\ veh/sec)(60\ sec)}{(.528\ veh/sec - .150\ veh/\text{sec})}$$

$$g_s = 31.2\ sec$$

Step 5. Determine the total delay. The total delay can be calculated as the area under the curve in the cumulative vehicle diagram or queue accumulation polygon. Here we will use the queue accumulation polygon, divided into six separate polygons to facilitate the calculation of the area as shown in Figure 7.

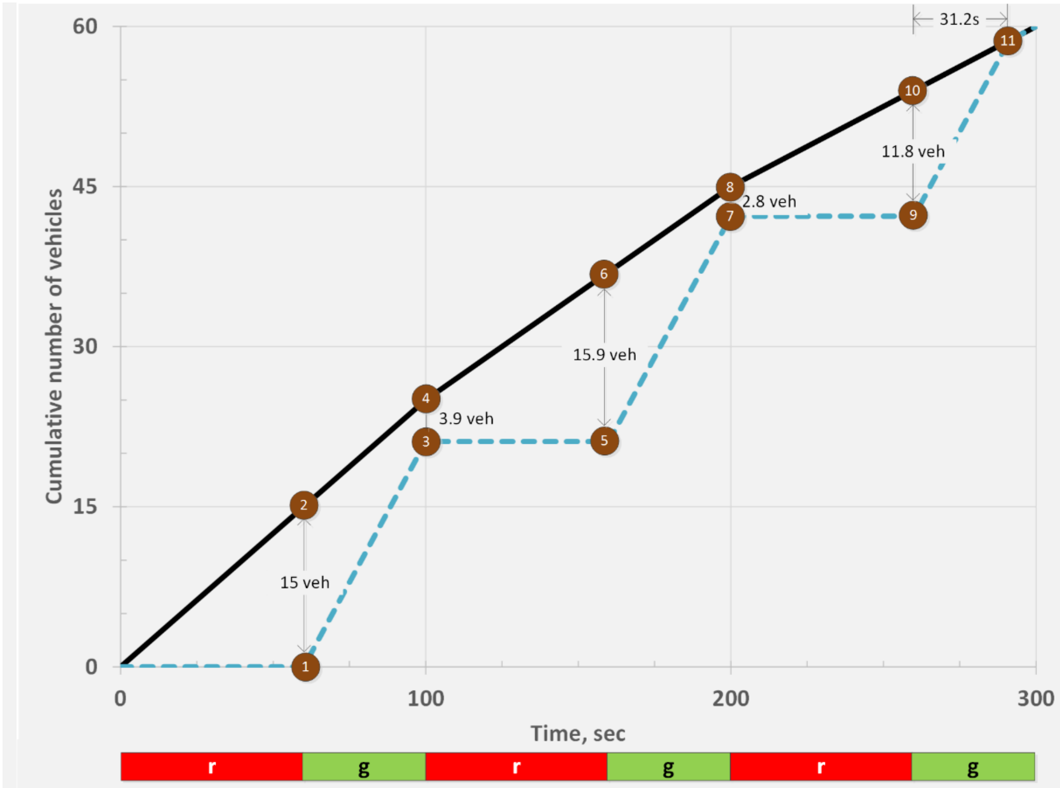

Figure 5. Cumulative vehicle diagram for Example 14

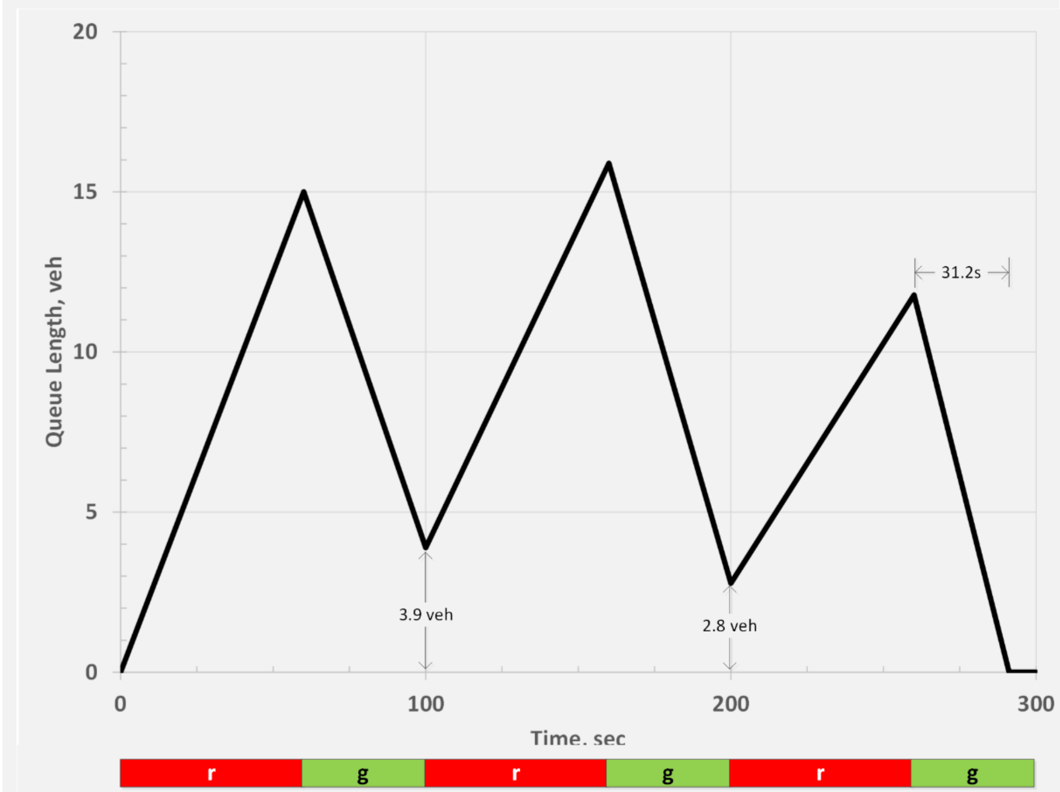

Figure 6. Queue accumulation polygon for Example 14

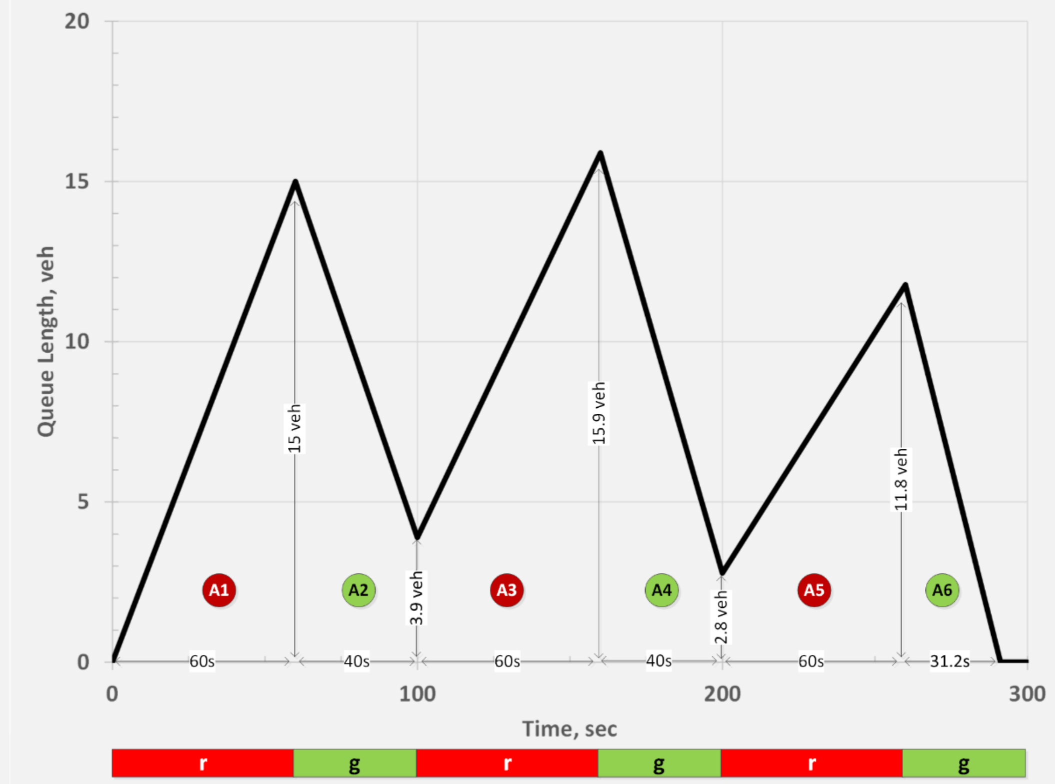

Figure 7. Constituent polygons from queue accumulation polygon

The calculations to determine the areas of each of the six constituent polygons are shown below.

Cycle 1:

$$A1 = \frac{15\ veh}{2} \times (60\ sec) = 450\ veh - sec$$

$$A2 = \frac{3.9\ veh + 15\ veh}{2} \times (100\ sec - 60\ sec) = 378\ veh - sec$$

Cycle 2:

$$A3 = \frac{15.9\ veh + 3.9\ veh}{2} \times (160\ sec - 100\ sec) = 594\ veh - sec$$

$$A4 = \frac{2.8\ veh + 15.9\ veh}{2} \times (200\ sec - 160\ sec) = 374\ veh - sec$$

Cycle 3:

$$A5 = \frac{11.8\ veh + 2.8\ veh}{2} \times (260\ sec - 200\ sec) = 438\ veh - sec$$

$$A6 = \frac{11.8\ veh}{2} \times (291.2\ sec - 260\ sec) = 184\ veh - sec$$

The sum of these areas is the total delay for three cycles.

$$D_t = A_1 + A_2 + A_3 + A_4 + A_5 + A_6$$

$$D_t = 2418\ veh - sec$$

Step 6. Determine the number of vehicles that arrive during the three cycles. Based on the arrival rates given in Table 1 and the length of the cycle (*C* = 100 sec), the total number of vehicles arriving during the three cycles is calculated below. Note that the number of vehicle arrivals can also be determined from the cumulative vehicle diagram in Figure 5.

Cycle 1:

$$Vehicle\ arrivals = (.25\ veh/sec)(100\ sec) = 25\ \ veh$$

Cycle 2:

$$Vehicle\ arrivals = (.20\ veh/sec)(100\ sec) = 20\ \ veh$$

Cycle 3:

$$Vehicle\ arrivals = (.15\ veh/sec)(100\ sec) = 15\ \ veh$$

The total arrivals during three cycles:

$$Vehicle\ arrivals = 25\ veh + 20\ veh + 15\ veh = 60\ \ veh$$

Step 7. Calculate average delay.

$$d_{avg} = \frac{D_t}{number\ vehicles\ arrived} = \frac{2418\ veh - sec}{60\ veh} = 40.3\ sec$$

7.2 Other Performance Measures for a Signalized Intersection

In addition to delay, other measures are used to estimate how well (or poorly) an intersection is operating. Two of these measures are the volume-to-capacity ratio and the back of queue. The volume-to-capacity ratio was discussed earlier in section 7 of this module.

The *back of queue* is the position of the vehicle stopped farthest from the stop line as a result of the red signal indication. It is useful for determining some of the geometric elements of the intersection, such as the length of a left turn bay. It is also useful in determining whether the operation of an upstream intersection will be affected by a spillback of the queue. The *back of queue size* (Q_{back}, the number of vehicles from the stop bar to the back of queue when the queue clears) is distinguished from the *maximum queue length* Q_{max}, the maximum number of vehicles in the queue, which occurs at the end of the effective red. These parameters are illustrated in Figure 8.

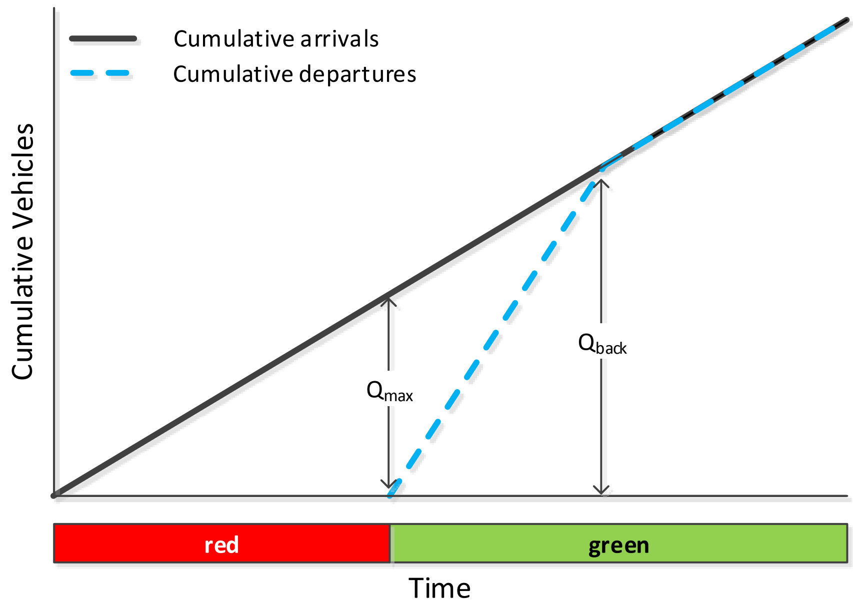

Figure 8. Maximum queue and back of queue size

The back of queue size is found by computing the product of the arrival rate and the sum of the effective red time and the queue service time, as given in Equation 8.

Equation 8

$$Q_{back} = v(r + g_s)$$

where

Q_{back} = back of queue size, veh,
v = arrival flow rate, veh/sec,
r = effective red time, sec,
g_s = queue service time, sec, and

Example 15. Back of Queue Size Calculation

The intersection shown in Figure 9 has an afternoon peak-hour volume of 250 veh/hr (.069 veh/sec) for the westbound left turn movement. The saturation flow rate is 1900 veh/hr (.528 veh/sec). The afternoon cycle length is 80 sec and the effective green for the left turn approach is 12 sec. Assume that vehicles maintain a 25 ft spacing while in queue (that is, the distance from the front of one vehicle to the front of the following vehicle is 25 ft). Calculate the back of queue size and determine whether the left turn bay of 125 feet is of sufficient length to accommodate this length of queue assuming that left turns are protected.

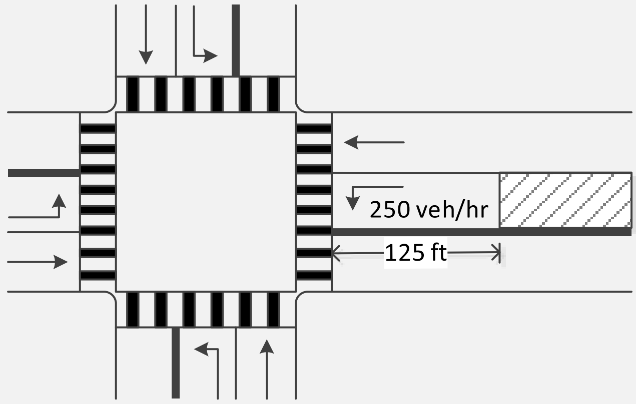

Figure 9. Intersection geometry and approach volume

Step 1. Calculate the queue service time using Equation 2.

$$g_s = \frac{vr}{s-v} = \frac{(.069\ veh/sec)(68\ \text{sec})}{(.528\ veh/sec - .069\ veh/sec)} = 10.3\ sec$$

Step 2. Calculate the back of queue size using Equation 8.

$$Q_{back} = v(r + g_s) = (.069\ veh/sec)(68\ \text{sec} + 10.3\ \text{sec}) = 5.4\ veh$$

Step 3. Calculate the distance upstream from the intersection stop bar reached by the back of queue.
Given the average vehicle spacing of 25 ft (and assuming 6 vehicles, closer to reality than the estimate of 5.4 vehicles), the distance upstream from the stop bar to the back of queue is:

$$distance = (6\ veh)(25\ ft/veh) = 150\ ft$$

Step 4. Compare the distance reached by the back of queue to the storage available in the left turn bay.
The distance reached by the queue is 150 ft upstream of the stop bar. But the space available in the storage bay is 125 ft. There is not sufficient space available in the left turn bay to store the last (sixth) vehicle in the queue. This could result in a left turn vehicle blocking the westbound through movement, which could be both capacity problem and a safety problem.

7.3 Level of Service

The Highway Capacity Manual defines *quality of service* as how well a transportation facility operates from the perspective of the users of that facility. *Level of service* is a "quantitative stratification of a performance measure that represents the quality of service." For a signalized intersection, average delay is used as the performance measure. The HCM provides level of service ranges for a signalized intersection as shown in Table 2. These ranges can be applied to an intersection approach or to the entire intersection. It should be noted that if the volume-to-capacity ratio exceeds one, the level of service will be F, regardless of the estimated delay.

Table 2. Level of service ranges for a signalized intersection

Level of Service	Average control delay per vehicle (sec)
A	≤ 10
B	> 10 and ≤20
C	> 20 and ≤35
D	> 35 and ≤55
E	> 55 and ≤80
F	> 80

Example 16. Estimation Of Intersection Delay And Level Of Service

The demand and delay for four approaches of a signalized intersection are shown in Table 3. Determine the level of service for each approach and for the intersection as a whole.

Table 3. Traffic volume and delay for intersection approaches

Approach	Volume (veh/hr)	Volume (veh/sec)	Delay (sec)
Northbound	650	.181	25
Southbound	850	.236	18
Eastbound	200	.056	60
Westbound	300	.083	50

Step 1. Determine the level of service for each approach.
Based on the level of service ranges provided in Table 2, the level of service for each approach is shown in Table 4.

Table 4. Traffic volume and delay for intersection approaches

Approach	Volume (veh/hr)	Volume (veh/sec)	Delay (sec)	LOS
Northbound	650	.181	25	C
Southbound	850	.236	18	B
Eastbound	200	.056	60	E
Westbound	300	.083	50	D

Step 2. Determine the level of service for the intersection.
The volume and delay data given in Table 4 for each approach are used to determine the delay and level of service. The elements of the numerator for Equation 7 are calculated first:

$$d_{NB}v_{NB} = (25\ sec)(.181\ veh/sec) = 4.5\ veh$$

$$d_{SB}v_{SB} = (18\ sec)(.236\ veh/\text{sec}) = 4.2\ veh$$

$$d_{EB}v_{EB} = (60\ sec)(.056\ veh/sec) = 3.4\ veh$$

$$d_{EB}v_{EB} = (50\ sec)(.083\ veh/sec) = 4.2\ veh$$

Then, using Equation 7, the average delay for the intersection is:

$$d_{int} = \frac{\sum d_i v_i}{\sum v_i}$$

$$= \frac{(4.5 + 4.2 + 3.4 + 4.2)}{(.181 + .236 + .056 + .083)\ veh/sec} = 29.3\ sec$$

An average delay of 29.3 sec corresponds to an intersection level of service of C. This is acceptable. However, the level of service on the eastbound approach is E, nearly unacceptable. The westbound approach also experiences significant delay, at level of service D. The operation of the intersection may be improved if the signal timing were altered so that the minor approaches had more green time. The topic of signal timing is covered in the section 9.

7.4 Summary of Section 7

What You Should Know and Be Able to Do:

- Calculate uniform delay at a signalized intersection
- Describe and apply measures of effectiveness for a signalized intersection
- Describe and apply the level of service framework

Concepts You Should Understand:

- Concept 7.1: Queue service time

The queue service time is the ratio of the length of the queue at the beginning of the effective green time and the rate at which the queue clears after the beginning of the effective green.

- Concept 7.2: Uniform delay and the queue accumulation polygon

The total uniform delay for one movement during one cycle can be represented as the area of the queue accumulation polygon. The average uniform delay is the total uniform delay divided by the number of vehicles that arrive during the cycle.