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REVIEW OF INTERNATIONAL PRACTICES USED TO EVALUATE UNSIGNALIZED INTERSECTIONS

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BACKGROUND

The design of unsignalized intersections has been given different importance in different countries. In many countries, for instance in Australia and Germany, 4-leg- and T-intersections are designed to meet safety standards, not operational standards. Consequently there is a requirement to ensure that adequate sight distance is available. Under German conditions, minor road approaches have been restricted to a one-lane approach, to avoid mutual sight obstruction by cars. In these cases, it would not be expected that the capacity would be calculated when these unsignalized intersections are constructed. However, with increasing traffic volumes, the maximum capacity of these intersections is becoming more and more important.

Therefore, particularly in a number of European countries, a sophisticated series of procedures has been developed for the evaluation of capacity and traffic flow quality at unsignalized intersections. Previously, these procedures have mainly been used as a warrant for installing a traffic signal at an intersection. More recently, these procedures are also being used in the network design process. When used in connection with other traffic engineering capacity estimation procedures (e.g. for signalized intersections), they serve to find strategies for traffic guidance in networks which help to avoid the installation of traffic signals and reduce delays.

Roundabouts, although unsignalized, should be considered differently from the cross- and T-intersections. Some of the techniques used in the analysis of roundabouts can also be used in the appraisal of signalized intersections (cf. Brilon, Stuwe, 1993; Troutbeck, 1993).

For uncontrolled intersections, we find the rule "right before left" or "right-of-way" (when driving on the right side of the street) in many countries (especially on the European continent, based on the early French highway codes). A method of capacity estimation for this type of operation has been given by Vasarhely (1976). Several authors have found that this basic rule of priority has neither a positive influence on safety nor on capacity. For capacity, the alternative rule "priority to the left" would be more advantageous (Hondermarq, 1968). To some extent, the North American All-way-stop-controlled intersections (AWSC) operates quite similar. A complete procedure of capacity calculation for this type has been given in TRB *Circular No. 373* (TRB, 1991) which is based on Kyte's research (Kyte, 1989).

An approximation of overall daily capacity of intersections is as follows:

1. uncontrolled (priority to the right)	1000—1500	pcu/d
2. yield or stop sign controlled	5000—12000	pcu/d
3. rotaries/roundabouts		
single lane	20000—28000	pcu/d
multilane	35000— ?[1]	pcu/d
4. signalized	20000—80000[2]	pcu/d

[1] Varies from country to country
[2] Depending on the no. of lanes for the different movements

This *Circular* concentrates on group 2, i.e. at-grade cross or T-intersections with one main street having priority over movements from the minor street. The subordination of the nonpriority streams is indicated by a yield or stop sign, the American equivalent of which is 2-way-stop-controlled intersection (TWSC).

The *Circular* discusses the differences between the analysis techniques for TWSC intersections used in different countries of the world. The practices are quite different and have different origins. Before discussing the practices of individual countries, it is worthwhile to discuss the desirable attributes of an analysis procedure.

Basically, we must distinguish between 2 different types of solutions for the analysis of TWSC traffic operations:

1. Gap acceptance theory.
2. Empirical regression technique.

Unsignalized intersections give no positive indication or control to the driver. He/she is not told when he/she should enter the intersection. The driver alone must decide when it is safe to enter the intersection. Therefore, the driver must perceive distances and velocities of other road users and he/she must have a feeling for his/her own car's performance. This process of perception and decision is described by the group of gap acceptance models. Based on quite simple definitions of critical gaps, we find a huge number of different models and solutions. These types of solutions are closely related to queueing theory.

All analysis procedures have relied on gap acceptance theory to some extent or they have understood that the theory is the basis for the operation even if they have not used the theory explicitly.

Although gap acceptance is generally well understood, it is useful to consider the gap acceptance process as one that has three basic elements (Troutbeck, 1991).

1. First is the extent to which drivers find the gaps or opportunities of a particular size useful when attempting to enter the intersection.

2. Second is the proportion of gaps of a particular size in the priority stream that are offered to the entering driver. The pattern of the inter-arrival times is also important.

3. A third requirement at unsignalized intersections is the interaction between streams.

These 3 items are discussed below. For an easy understanding of items 1 and 2 , it is useful to concentrate on the simplest case of an unsignalized intersection first (Figure 1).

All gap acceptance methods for unsignalized intersections are derived from a simple queueing model in which the crossing of two traffic streams is considered (see Figure 1). A priority traffic stream (major stream) of the volume q_p (veh/h) and a nonpriority traffic stream (minor stream) of the volume q_n (veh/h) are involved in this queueing model. Vehicles from the major stream can cross the conflict area without any delay. Vehicles from the minor stream are only allowed to enter the conflict area, if the next vehicle from the major stream is still t_c seconds away (t_c = critical gap), otherwise they have to wait. Moreover, vehicles from the minor stream can only enter the intersection t_f seconds after the departure of the previous vehicle (t_f = follow-on time or move-up time).

USEFULNESS OF THE GAPS

The behavior of minor street vehicles is normally represented by the 2 variables t_c and t_f.

t_c = critical gap = minimum time gap in the priority stream which a minor street driver is ready to accept for crossing or entering the major stream.

t_f = follow-on time = time gap between 2 successive vehicles from the minor street while entering the conflict area of the intersection.

It is obvious that t_c and t_f differ from driver to driver, from time to time and between intersections, types of movements and traffic situations. It is, therefore, necessary to define some type of representative characteristics to model the usual behavior of drivers.

Speaking more mathematically, it is generally assumed in the theory that drivers are both consistent and homogeneous. Consistent drivers are expected to behave the same way every time at all similar situations. The population of drivers are also expected to be homogeneous in that all drivers behave in exactly the same way at any location. This theory suggests that if a driver has a critical gap of t_c = 4 s, no driver will enter the intersection unless the gap between the priority stream vehicles is at least the "critical acceptance gap" or simply the "critical gap". If, however, a gap in the main stream of more than 4 s is provided, the minor street driver will depart. He/she will require the same 4 s at

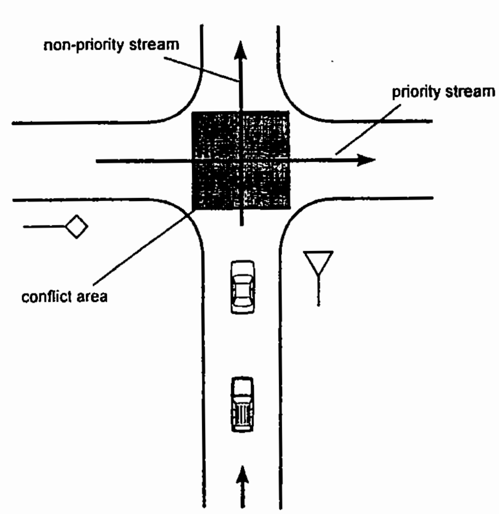


FIGURE 1 Illustration of the basic queueing system.

all other times he/she approaches the same intersection and so will all other drivers at that intersection.

Within the gap acceptance theory, it is further assumed that a number of drivers will be able to enter in very long gaps. Usually, the minor stream vehicles (those yielding right of way) enter in long gaps at headways of the follow-on time, t_f.

Note that other researchers have used a different concept for the critical gap and the follow-on time. For example, McDonald, Armitage (1978) and Siegloch (1973) independently described a concept where a lost time, or a "zero-gap" time is subtracted from each major stream gap and the remaining time is considered 'usable'. This 'usable' time divided by the saturation headway gives an estimate of the absorption capacity of the minor stream. However, the results from this concept are almost identical to those from techniques using the conventional t_c and t_f definitions.

The assumptions for the gap acceptance approach are clearly not realistic. If drivers were heterogeneous, with each driver having a different t_c and t_f value from realistic distributions, then the entry capacity would be decreased. However, if drivers are inconsistent, with a driver capable of accepting a shorter gap than those rejected, then the capacity would be increased. If drivers are assumed to be both consistent and homogeneous, rather than more realistically inconsistent and heterogeneous, then the difference in the predictions is only a few percent. That is, the overall effect of assuming that drivers are consistent and homogeneous is minimal and for simplicity, consistent and homogeneous driver behavior is assumed (see Grossmann, 1991 and Troutbeck, 1988).

It has been found that the gap acceptance parameters t_c and t_f are affected by the speed of the major stream traffic (Harders, 1976, and Troutbeck, 1988). It is also expected that drivers are influenced by the difficulty of the maneuver. The more difficult a maneuver, the longer the gap acceptance parameters. There has also been a suggestion that drivers require a different critical gap when crossing different streams within the same maneuver. For instance a turn movement across a number of different streams may require a driver having a different critical gap or time period between vehicles in each stream (Fisk, 1989). This is seen as an unnecessary complication given the other variables to be considered.

For the estimation of critical gaps t_c and follow-on times t_f from observations, a long series of methods has been proposed. A good overview was given by Miller (1972).

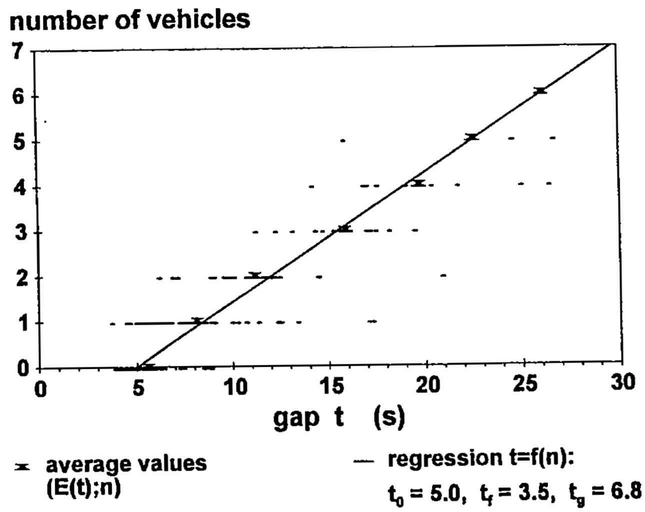


FIGURE 2 Evaluation of critical gaps and follow-on time according to Siegloch's method (fig. obtained from Brilon, Grossmann, 1991).

For the estimation of t_c and t_f from saturated conditions, i.e. continuous queueing on the minor street, Siegloch's proposal for t_c and t_f estimation is quite simple and reliable (Siegloch, 1973). His procedure is as follows (cf. Figure 2):

- Observe a traffic situation during times when there is, without interruption, at least one vehicle queueing in the minor street.
- Record the number of vehicles, "i", entering each main stream gap of duration "t".
- For each of the gaps accepted by "i" drivers: compute the average of the accepted gaps (O in Figure 2).
- Find the linear regression of these averages (average gap as a function of i).
- The increase of this regression line from i to i+1 is t_f.
- The intersection of the regression line with the horizontal axis gives: $t_0 = t_c - t_{f/2}$

The Siegloch method is easy to apply and it is reliable, since the way to estimate critical gap and follow-on time is exactly compatible with the derivation of the corresponding capacity formula (eq. 2.4.2). However, the method is only suitable under oversaturated conditions. On the other hand, traffic operation with undersaturated conditions can also provide information about t_c and t_f. Here, however, these procedures are not easy.

Only the follow-on time t_f can easily be observed directly. For an evaluation, the times between vehicles from the minor street entering the same gap of the priority stream should be measured. As an instrument, a normal stopwatch or, better, a video camera with time indicated in each frame could be used. The average time between successive minor street vehicles gives a good estimate of the follow-on time t_f. For practical purposes, at least n_f observations should be used to get an estimate of sufficient reliability (S % probability that the estimate is in a range of r_f around the true estimate).

With $\delta_f \approx 0.4 \cdot t_f$ (Harders, 1976), we get from sampling theory:

$$n_f = a_f \cdot \frac{1}{r_f^2} \qquad (2.1.1)$$

with

n_f	= necessary no. of observed follow-on times	(-)
r_f	= relative error = e_f/t_f	(-)
e_f	= absolute error	(s)
δ_f	= standard deviation of the statistical distribution of the t_f	(s)
a_f	= function of S (S = level of confidence):	(-)

S	90%	95%	99%
a_f	0.4	0.6	1

It is more complicated to estimate the critical gap t_c from traffic observations with undersaturated conditions, since a critical gap cannot be observed directly. If we observe a driver on the minor street, we can state: his/her t_c is greater than the maximum rejected gap and t_c is smaller than the gap he/she accepts. This is true if the driver behaves consistently (see above).

If we observe a series of accepted gaps t_a (=gaps in the priority stream accepted by minor street vehicles), then these accepted gaps can be described by an empirical statistical distribution function $F_a(t)$ (cf. Figure 3). Then the distribution function $F_c(t)$ of critical gaps t_c must be left of the t_a distribution.

Under the assumption of:

a) Exponentially distributed priority stream gaps, and
b) Normal distribution for t_a and t_c,

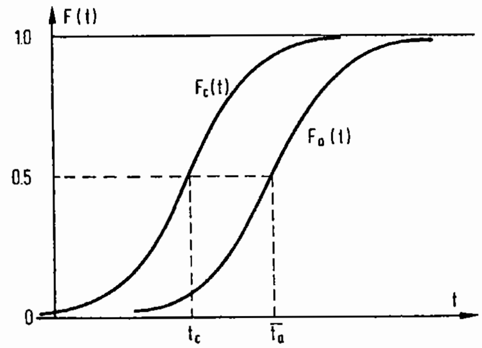


FIGURE 3 Cumulative distribution function of accepted (=utilized) gaps (F_a) and critical gaps (F_c)

Ashworth (1968, 1970) found that the average critical gap t_c can be estimated from t_a (= mean of the accepted gaps t_a; in s) by

$$t_c = \bar{t}_a - p \cdot s_a^2 \qquad (s) \qquad (2.1.2)$$

with

$\bar{t}_a$	= average of the accepted gaps	(s)
p	= priority traffic volume	(veh/s)
s_a^2	= variance of the distribution of t_a	(s^2)

For cases of more realistic conditions instead of assumption a) and b) (see above), Hewitt (1985, 1993) found a procedure for estimating t_c.

The maximum likelihood method calculates the probability of the critical gap being between the largest rejected gap, r_i, and the accepted gap, a_i. To estimate this probability for each driver, the user must specify the general form of the distribution of the critical gaps for the population of drivers and then to assume that all drivers are consistent. The likelihood that the drivers critical gap will be between a_i and r_i is given by $F(a_i)$ - $F(r_i)$. The likelihood is then summed for all n drivers to give:

$$\prod_{i=1}^{n} [F(a_i) - F(r_i)] \qquad (2.1.3)$$

where there are n drivers and:

F(x) is the cumulative probability distribution of the critical gaps;
a_i is the accepted gap for driver i; and
r_i is the largest rejected gap for driver i.

The logarithm of the likelihood, L, is given by

$$L = \sum_{i=1}^{n} \ln[F(a_i) - F(r_i)] \tag{2.1.4}$$

The likelihood is maximized when the logarithm of the likelihood is also maximized. Appropriate values for the critical gap distribution parameters (the mean and the variance) are found by setting the partial derivatives of L with respect to these parameters, to zero.

In practice, the log-normal distribution is often used as the distributation of the critical gaps as well as for the a_i and r_i. The equations that need to be solved by iteration are then

$$\sum_{i=1}^{n} \frac{[f(r_i) - f(a_i)]}{[F(a_i) - F(r_i)]} = 0 \tag{2.1.5}$$

and

$$\sum_{i=1}^{n} \frac{[(r_i - \mu)f(r_i) - (a_i - \mu)f(a_i)]}{[F(a_i) - F(r_i)]} = 0 \tag{2.1.6}$$

where

μ is the mean of the logarithms; and
$f(x)$ is the probability density function of the critical gap distribution.

The mean critical gap has been found to be an acceptable quantity. Troutbeck (1992) describes a procedure for estimating the critical gap parameters using this maximum likelihood technique.

Finally, some general remarks should be made:

t_c and t_f are predetermined by physical and dynamic relations of vehicle movements to a considerable degree. Therefore, t_c and t_f depend on the geometric layout of an intersection. Due to different design standards, the geometric design can differ systematically between countries. In addition to this, t_c and t_f represent driver behavior. Since this depends on traffic rules and general attitudes of drivers, these values should also differ between countries.

Estimations of the t_c and t_f for different countries can be obtained from the sources mentioned in Table 1.

The t_c and t_f values mentioned up to this point only apply for passenger cars. Trucks and buses, of course, have greater values. This can be introduced into calculations by converting these larger vehicles into passenger car units (pcu). For this, the shared-lane formula (eq. 2.9.1) can be used when t_c and t_f values have been measured for heavy vehicles. The application of this technique enabled pcu-equivalents for trucks to be estimated. For example, the German values that were estimated for the first time by Jessen (1968) and included in the HCM (1985, table 10-1) as well, have later been verified based on Harders' (1976) measurements of t_c values. They indicate for level minor street approaches:

truck, bus	1.5 pcu
truck+trailer	2.0 pcu
motorcycle	0.5 pcu.

For upgrades and downgrades on minor street entries, these values should be modified (cf. HCM 1985, table 10-1).

There is still one other influence of t_c and t_f which has to be mentioned. This is the dependence of these terms on traffic volumes in both the priority and nonpriority streams. When there are heavier traffic volumes, drivers have been observed to accept shorter critical gaps (Troutbeck, 1989 and Kyte e.a., 1991). This could be due to either drivers adjusting their critical gap parameters with the time spent in the queue, or they may simply assess some intersections as being more demanding and consequently adopt shorter critical gap parameters that are independent of their waiting time. Each approach gives similar estimates for the performance of the intersection.

DISTRIBUTION OF GAP SIZES IN PRIORITY STREAMS

The distribution of gaps between the vehicles in the different streams - especially in priority streams - has a major effect on the performance of the unsignalized intersection. However, here we need only look at the distribution of the larger gaps, those that are likely to be accepted. There is no point in modelling the shorter gaps which are expected to be rejected.

The usual model is to use a random arrival pattern. That is that the interarrival times follow an exponential distribution. The cumulative headway distribution, F(t), is used to describe the distribution. F(t) is the probability of an interarrival time or gap being less than t, and is given by:

$$F(t) = 1 - e^{-pt} \tag{2.2.1}$$

where p is the traffic flow in veh/s.

TABLE 1: A SELECTION OF CRITICAL GAP VALUES FROM DIFFERENT COUNTRIES

	Reference	t_c	t_f	reliability[1]
Australia	Aust. Roads, 1986	4–8	2–5	2
Czechoslovakia	Jirava, Karlicky, 1988	5.0–8.5	—	
France	Lassare e.a., 1991	3.2–7.3	—	
Germany	Harders, 1976[2]	4.5–9.6	1.7–5.9	5
Poland	Tracz, 1991	4.8–7.7	—	
Sweden	Bang e.a., 1978	3.5–7.5	2.2–4.2	4
USA	HCM 1985; table 10.2	5.0–8.5	$0.6 * t_c$	5
	Fricker e.a., 1991	4.7–5.8	—	2

Remarks:

[1] reliability on a scale from:
1 = rough estimation based on one single observation
to 10 = representative for the whole country; based on thousands of observations at many intersections; evaluated by theoretically correct methods; actual to the present (1993) situation.

[2] extremely dependent on main street average velocity.

A better model is one that uses a dichotomized distribution like Cowan's (1975) M3 model or Schuhl (1955). These models assume that there is a proportion of vehicles that are free of interactions and travel at larger headways. These vehicles are termed "free" vehicles. The remaining vehicles travel at short headways and in bunches or platoons. In Cowan's M3 model, the proportion of free vehicles is α and the vehicles travelling in bunches are assumed to be at headways of only t_m. The proportion of bunched vehicles is $1-\alpha$. Of course, α depends on traffic volumes. One proposal for this relation is (cf. Brilon, 1988b)

$$\alpha = e^{-xp} \qquad (2.2.2)$$

x = const., between 6 and 9 depending on the type of street.

Other linear functions for α have been included in Tanner's (1962) model and in Troutbeck (1989).

The equation for the cumulative probability distribution for Cowan's model is:

$$F(t) = 1 - \alpha e^{-s(t-t_m)} \quad for\ t>t_m \qquad (2.2.3)$$

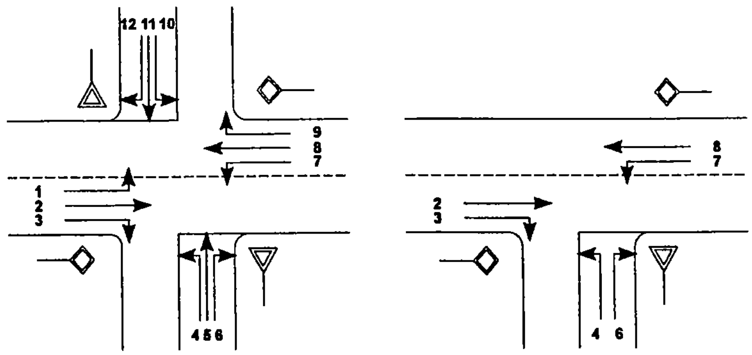
and

$$F(t) = 0 \qquad \text{otherwise.}$$

where s is a decay constant given by the equation

$$s = \frac{\alpha \cdot P}{(1 - t_m \cdot P)} \quad \text{(veh/s)} \qquad (2.2.4)$$

It is usual to consider the headways between successive vehicles to be independent. This then leads to a geometric distribution of bunch size. Another distribution of bunches used in the analysis of unsignalized intersections is the Borel-Tanner distribution (Tanner, 1961). For the same mean bunch size, the Borel-Tanner distribution has a larger variance. This latter distribution generally predicts a greater number of longer bunches than does the geometric distribution.



rank 1: 2,3,8,9
2: 1,7,6,12
3: 5,11
4: 4,10

rank 1: 2,3,8
2: 7,6
3: 4

FIGURE 4 Traffic streams and their rank of priority. The numbers beside the arrows indicate the enumeration of streams given by the German guidelines (FGSV, 1991).

HIERARCHY OF TRAFFIC STREAMS AT A TWSC INTERSECTION

At all unsignalized intersections except roundabouts, there is a hierarchy of streams. Some streams have absolute priority (rank 1), whilst others have to give way to higher order streams. In some cases, streams have to give way to some streams which in turn have to give way to others. It is useful to consider the streams as having different levels of priority. These different ranks of priority are established by traffic rules. For instance:

- Priority 1 stream (rank 1) has absolute priority and does not need to yield right-of-way to another stream;
- Priority 2 stream (rank 2) has to give way to a priority 1 stream;
- Priority 3 stream (rank 3) has to give way to a priority 2 stream and in turn to a priority 1 stream; and
- Priority 4 stream (rank 4) has to give way to a priority 3 stream and in turn to priority 2 and 1 streams.

This is illustrated in Figure 4 produced for traffic on the right side. The figure illustrates that the left turners on the major road have to give way to the through traffic on the major road. The left turning traffic from the minor road has to give way to all other streams but is also affected by the queueing traffic in the priority 2 stream.

CAPACITY

Based on the gap acceptance model, the capacity of the simple 2-stream situation (cf. Figure 1) can be evaluated by elementary probability theory methods if we assume

a) Constant t_c and t_f values;
b) Exponential distribution for priority stream gaps (cf. eq. 2.1);
c) Constant traffic volumes for each traffic stream.

Assumption (a) implies that gaps greater than or equal to $t_c + (i-1)t_f$ but less than $t_c + i * t_f$ will be accepted by i stream 2 drivers. From these assumptions, the maximum entry flow ($q_{e,max}$) or capacity is given by

$$q_{e,max} = q_p \cdot \frac{e^{-p t_c}}{1 - e^{-p t_f}} \quad \text{(veh/h) (2.4.1)}$$

p	$= q_p / 3600$	(veh/s)
q_p	= volume of priority stream	(veh/h)

(See Harders, 1968, for derivation)

Some authors, notably Siegloch (1973) and McDonald and Armitage (1978), have revised the first assumption by assuming that a gap of size t will be accepted by $(t-t_o)/t_f$ drivers. This quantity is not necessarily an integer and is based on a linear approximation of the step function assumed above.

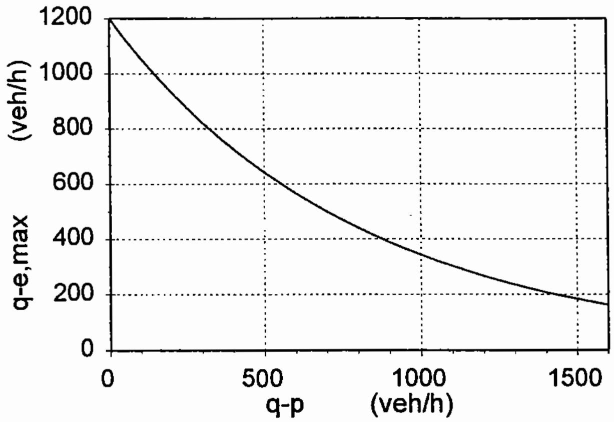


FIGURE 5 Shape of the relation between capacity ($q_{e,max}$) and priority street volume (q_p) for the two-stream situation (cf. fig. 1) based on eq. 2.4.2 with t_c = 6 s and t_f = 3 s.

Based on this assumption Siegloch's (1973) equation for capacity is:

$$q_{e,max} = \frac{3600}{t_f} \cdot e^{-p \cdot t_o} \quad \text{(veh/h)} \quad (2.4.2)$$

$$t_o = t_c - t_f/2$$

These formulae result in a capacity-conflicting flow curve shown in Figure 5.

These formulae (2.4.1 and 2.4.2) produce slightly different results due to the slight revision in assumption (a). However, the magnitude of these differences is negligible for practical purposes.

The idealized assumptions a) through c), however, are not realistic. Therefore, different attempts to drop one or the other assumption have been made. Siegloch (1973) studied different types of gap distributions for the priority stream (cf. Brilon, 1988b, Figure 4) based on analytical methods. Similar studies have also been performed by Troutbeck (1986). Grossmann (1991) investigated these effects by simulations. Each study showed:

- If the constant t_c and t_f values are replaced by realistic distributions (cf. Grossmann, 1988), we get a decrease in capacity.
- If the exponential distribution of major stream gaps is replaced by more realistic headway distributions, we get an increase in capacity of about the same order of magnitude as the effect of using the more realistic distributions for t_c and t_f values (Grossmann, 1991, and Troutbeck, 1986).

There is then little to be gained from using complicated headway distribution.

More general solutions have been obtained by replacing the exponential headway distribution used in assumption (b) with a more realistic one. This more general equation is:

$$q_{e,max} = \frac{\alpha q_p e^{-s(t_c - t_m)}}{1 - e^{-s t_f}} \quad \text{(veh/h)}$$

where

$$s = \frac{\alpha \cdot p}{1 - t_m \cdot p} \quad \text{(veh/s)} \quad (2.4.3)$$

with t_m = headway between bunched vehicles (s)
α = proportion of free vehicles (s)

This is similar to equations reported by Tanner (1967), Gipps (1982), Troutbeck (1986), Cowan (1987) and others. If α is set to 1 and t_m to 0, then Harders' equation is obtained. If α is set to $1\text{-}pt_m$, then this equation reduces to Tanner's (1962) equation.

If the revised relationship of a linear function between the number of drivers entering and the size of gap is used, then the associated capacity equation is

$$q_{e,max} = \frac{\alpha q_p e^{-s(t_o - t_m)}}{s \cdot t_f} \quad \text{(veh/h)} \quad (2.4.4)$$

or

$$q_{e,max} = \frac{3600(1 - t_m p) e^{-s(t_o - t_m)}}{t_f} \quad \text{(veh/h)}$$

where $t_o = t_c - 1/2\, t_f$ (s)

This was proposed by Jacobs (see Brilon, 1988b, for details).

A complete analytical solution for a realistic replacement of assumptions a) and b) within the same set of formulae is given by Wegmann (1991).

Since this solution is complicated, many researchers have tried to find realistic capacity estimations by simulation studies. This applies especially for the new German method (FGSV, 1991) and the Polish method.

If there is more than one lane of traffic on the priority stream, then the headway distribution approaches an exponential distribution as the number of lanes increases. A generalized form of the capacity equation was given by Tanner (1967) and later extended by Gipps (1982), Troutbeck (1986) and Fisk (1989). The equation is:

$$q_{e,max} = \frac{s'\prod_{i=1}^{n}(1-t_m p_i)e^{-s'(t_c-t_m)}}{1-e^{-s't_f}} \cdot 3600 \quad \text{(veh/h)} \quad (2.4.5)$$

where

$$s' = \sum_{i=1}^{n} s_i$$

$$s_i = \frac{\alpha_i \cdot p_i}{1-p_i \cdot t_m}$$

n is the number of lanes

QUALITY OF TRAFFIC OPERATIONS/QUEUEING THEORY

In general, the performance of traffic operations at an intersection can be represented by these variables (measures of effectiveness, MOE):

1. Average delay;
2. Average queue lengths;
3. Distribution of delays;
4. Distribution of queue lengths (i.e. no. of veh. queueing on the minor road);
5. No. of stopped vehicles and no. of accelerations from stop to normal velocity; and
6. Probability of the empty system (p_o).

Distributions can be represented by:

- Standard deviations.
- Percentiles.
- The whole distribution.

To evaluate these measures, two tools can be used to solve the problems of gap acceptance:

- Queueing theory.
- Simulation.

Each of these MOEs are a function of q_p and q_n, the proportion of "free" vehicles and the distribution of bunch size length.

Solutions from queueing theory in the first step concentrate on average delays.

A general form of the equation for the average delay per vehicle is:

$$D = D_{min}\left[1 + \frac{\gamma + \epsilon \cdot x}{1 - x}\right] \quad \text{(s)} \quad (2.5.1)$$

where

γ and ϵ are constants (–)
x is the degree of saturation
$= q_n/q_{e,max}$ (-)

and D_{min} has been termed Adams' delay after Adams (1936). Adams' delay is the average delay to the minor stream vehicles when the minor stream flow is very low. It is also the minimum average delay experienced by the minor stream vehicles.

Troutbeck (1990a) gives equations for γ, ϵ and D_{min} based on the formulations by Cowan (1987). If the minor stream vehicles are assumed to arrive at random, then γ is equal to 0. On the other hand, if there is bunching in the minor stream, then γ is greater than 0.

For random arrivals in the minor stream, ϵ is given by:

$$\epsilon = \frac{e^{qt_f} - qt_f - 1 + q(e^{qt_f}-1)D_{min}}{q(e^{qt_f}-1)D_{min}} \quad (–) \quad (2.5.2)$$

Note that ϵ is approximately equal to 1.0. D_{min} depends on the bunching characteristics in the major stream. If the bunch size distribution is geometric, then:

$$D_{min} = \frac{e^{s(t_c-t_m)}}{\alpha p} - t_f - \frac{1}{s} + \frac{st_m^2 - 2t_m + 2t_m\alpha}{2(t_m s + \alpha)} \quad \text{(s)}$$

(Troutbeck, 1986) (2.5.3)

t_m α : see equation 2.4.3.

Tanner's (1962) model has a different equation for Adams' delay, because the bunch size distribution in the major stream has a Borel-Tanner distribution. This equation is:

$$D_{min} = \frac{e^{-p(t_c-t_m)}}{(1-t_m p)p} - t_f - \frac{1}{p} + \frac{pt_m^2(2t_m p-1)}{2(1-t_m p)^2} \quad \text{(s)} \quad (2.5.4)$$

Another solution for average delay has been given by Harders (1968). It is not based on a completely sophisticated queueing theory. However, as a first approximation, this equation is quite useful:

$$D = \frac{1-\nu}{q_{e,max} - q_n} \cdot 3600 \quad (s) \quad (2.5.5)$$

= estimation of average delay to nonpriority vehicles

with

$\nu = e^{-(pt_c + nt_f)}$ (-)

$p = q_p / 3600;$ (veh/s)

$n = q_n / 3600;$ (veh/s)

$q_{e,max}$ according to eq. (2.4.1). (veh/h)

M/G/1 System

A more sophisticated queueing theory model can be developed by the assumption that the simple two-streams system (Figure 1) can be represented by a M/G/1 queue. The service counter is identical with the first queueing position on the minor street. The input into the system is formed by the vehicles approaching from the minor street which are assumed to arrive at random, i.e. exponentially distributed arrival headways (cf. eq. 2.2.1) (i.e. "M"). The time spent in the first position of the queue is the service time. This service time is controlled by the priority stream, with an unknown service time distribution. The "G" is for a general service time. Finally, the "1" in M/G/1 stands for 1 service channel, i.e. 1 lane in the minor street.

For the M/G/1 queueing system, in general, the Pollaczek-Chintchine formula is valid for the average delay of customers in the queue

$$D_q = \frac{x * W(1 + C_w^2)}{2(1-x)} \quad (s) \quad (2.5.6)$$

with:

x = degree of saturation

$= \frac{q_n}{q_{e,max}}$

W = average service time (s)

here: average time a minor street vehicle spends in the first position of the queue near the intersection.

$C_w = \sqrt{S_w^2} / W$ (-)

= coefficient of variation of service times

S_w^2 = variance of service times (s^2)

The total average delay of minor street vehicles is then:

$$D = D_q + W$$

In general, the average service time for a single-channel queueing system is: 1/capacity. If we derive capacity from the formulas 2.4.1 until 2.4.5 and if we include the service time W into the total delay, we get:

$$D = \frac{1}{\mu} \left(1 + \frac{x}{1-x} \cdot C\right) \quad (2.5.7)$$

μ = service rate (veh/s)

$= q_{e,max}/3600$

($q_{e,max}$: see 2.4)

$C = \frac{1 + C_w^2}{2}$ (-)

Up to this point, the derivations are of general validity. The real problem now is to evaluate C_w^2.

The only extremes which can be stated are:

- Regular service: Each vehicle spends the same time in the first position.

$\rightarrow S_w^2 = 0 \quad \rightarrow C_w^2 = 0 \quad \rightarrow C = 0.5$

This gives the solution for the M/D/1 queue.

- Random service: The times vehicles spend in the first position are exponentially distributed.

$\rightarrow S_w = W \quad \rightarrow C_w^2 = 1 \quad \rightarrow C = 1$

This gives the solution for the M/M/1 queue.

Unfortunately, neither of these simple solutions applies exactly for the unsignalized intersection problem. However, as an approximation, some authors recommend the application of eq. 2.5.7 with C = 1.

Equation 2.5.1 can be further transformed to:

$$D = D_{min}(1+\gamma)\left[1+\frac{(\gamma+\epsilon)}{(1+\gamma)}\frac{x}{1-x}\right] \quad \text{(s)} \quad (2.5.8)$$

where ϵ and γ are documented in Troutbeck (1990).

This is similar to the Pollaczek-Chintchine formula (eq. 2.5.6). The randomness constant C is given by $(\gamma+\epsilon)/(1+\gamma)$ and the term $1/D_{min}(1+\gamma)$ can be considered to be an equivalent 'capacity' or 'service rate'. Both terms are a function of the critical gap parameters t_c and t_f and on the headway distributions. However, C, γ and ϵ values are not available for all conditions.

For the M/G/1 system as a general property, the probability p_0 of the empty queue is given by:

$$p_o = 1-x \quad (2.5.9)$$

This formula is of sufficient reality for practical use at unsignalized intersections.

M/G2/1 system

Different authors have found that the service time distribution in the queueing system is better described by 2 types of service times, each of which has a specific distribution:

1. W_1 = service time for vehicles entering the empty system, i.e. no other vehicle is queueing on the approach of the subject vehicle. W_1 is similar to D_{min} (eq. 2.5.1 and 2.5.3)

2. W_2 = service time for vehicles joining the queue when other vehicles are already queueing.

Again, in both cases, the service time is the time the vehicle spends waiting in the first position near the stop line.

The first ideas for this solution have been introduced by Kremser (1962, 1964) and in a comparable way by Tanner (1962) as well as by Yeo, Weesakul (1964).

Kremser derived solutions for the expectation as well as for the variance of W_1 and W_2 (cf. Brilon, 1988b, eq. 13 and 14, or Brilon e.a., 1991, eq. 5-7). Kremser gave a solution for the evaluation of the average delay. However, it is easier to understand from Yeo's (1962) formula:

$$D_q = \frac{n}{2}\left(\frac{E(W_1^2)-E(W_2^2)}{v} + \frac{E(W_2^2)}{y}\right) \quad \text{(s)} \quad (2.5.10)$$

= average delay of vehicles in the queue at higher positions than 1.

v = y + z.

y = $1 - n \cdot E(W_2)$.

z = $n \cdot E(W_1)$.

The probability p_o of the empty queue is:

$$P_o = \frac{y}{v} \quad (2.5.11)$$

The application of this formula shows that the differences against eq. 2.5.9 are quite small (< 0.03).

If we also include the service time (= time of minor street vehicles spent in the first position) into the total delay, we get (cf. Brilon, 1988b):

$$D = \frac{E(W_1)}{x} + \frac{n}{2} * \frac{Y \cdot E(W_1^2) + z \cdot E(W_2^2)}{v \cdot y} \quad (2.5.12)$$

$E(W_1)$ = expectation of W_1

$E(W_1^2)$ = expectation of $(W_1 * W_1)$

Kremser (1965) provided formulae for $E(W_1)$, $E(W_1^2)$, $E(W_2)$ and $E(W_2^2)$ in so far as he restricted the validity to the special case of $t_c = t_f$.

Daganzo (1977) gave an improved solution for $E(W_2)$ and $E(W_2^2)$ which again was extended by Poeschl (1983). These new formulae were able to overcome Kremser's (1964) restrictions. It can, however, be shown that Kremser's first approach (eq. 2.5.10 and 2.5.12 where W_1, W_2 can be obtained from eq. 13 and eq. 14 in Brilon, 1988) also gives quite reliable approximative results for t_c and t_f values which apply to TWSC intersections. This total set of formulae has two drawbacks:

- The formulae are so complicated that they are far away from being suitable for practice. The only imaginable application is the use in computer programs.
- Moreover, these formulae are only valid under assumptions a), b) and c) on page 11. That means, for practical purposes, the equations can only be regarded as approximations and they only apply for under-saturated conditions or steady state conditions.

The Swedish Road Capacity Manual (Hansson, Bergh 1988 and Hansson, 1987) used the technique of the M/G2/1 queue to predict the average delay at intersections, the average queue length and the distribution of bunches using the Pollaczek-Chintchine formula (equation 2.5.6). Hansson has indicated that a good approximation for the average service time in the G2 case is:

$$W = B \cdot s_q + (1-B) \cdot s_n \quad (s) \quad (2.5.13)$$

where:

B can be approximated by the degree of saturation, i.e.:

B ≈ x (cf. eq. 2.5.6)

The variance of the service times were assumed to be given by the variance ratio V^2 which is given by:

$$V^2 = 0.5\frac{s_q+s_n}{s_q} \quad (s^2) \quad (2.5.14)$$

s_q = $1/q_{e,max}$ (s)
= $E(W_2)$ in eq. 2.5.10

s_n = D_{min} (s)
= $E(W_1)$ in eq. 2.5.10

with V^2 being approximately equal to 1 for all but the higher flows. These equations were used to estimate the waiting time using eq. 2.5.6. The distribution of queue lengths was calculated using queueing theory equations.

Queue length

In each of these queueing theory approaches, the average queue length (L) can be calculated by Little's rule (Little, 1961):

$$L = q_n \cdot D \quad (2.5.15)$$

The distribution of queue length then often is assumed to be exponential. However, a more reliable derivation of the queue length distribution was given by Heidemann (1991). Wu (1993) has improved these solutions into a set of formulae and graphs which can also be used for practical application.

Time dependent solution

The solution given by the conventional queueing theory above is a steady state solution. It is the solution that can be expected for non-time-dependent traffic volumes after an infinitely long time, and it is only applicable when the degree of saturation "x" is less than 1.

In practical terms, this means: The results of steady state queueing theory are only useful approximations if (Morse, 1962):

$$T > \frac{1}{(\sqrt{\mu}-\sqrt{n})^2} \quad (2.5.16)$$

with

T = time of observation over which the average delay should be estimated. (s)

μ = service rate
= $q_{e,max}$ / 3600 (veh/s)

n = volume of the minor traffic stream
= (q_n / 3600) (veh/s)

The inequality means: T should be considerably greater than the expression on the right side. This inequation can only be applied if μ and n are nearly constant during time interval T.

If this condition is not fulfilled, time-dependent solutions should be used. Exact mathematical solutions for this problem are so complicated that they seem not to be very promising for practical application.

There is, however, a heuristic solution for the case of the peak hour effect (Kimber, Hollis, 1979). That means: traffic volumes are below capacity before and after the peak period of duration T. During the peak period itself, traffic volumes are greater than before and after that period. They may even exceed capacity. For this situation, the average delay during the peak period can be estimated as:

$$D = (D_1 + E + 1/\mu) \cdot 3600 \quad (s) \quad (2.5.17)$$

$$D_1 = \frac{1}{2}(\sqrt{F^2+G}-F)$$

$$F = \frac{1}{\mu_0-q_0}\left[\frac{T}{2}(\mu-q)\cdot y + C(y-\frac{h}{\mu})\right] + E$$

$$G = \frac{2T\cdot y}{\mu_0-q_0}\left[C\,\frac{q}{\mu} - (\mu-q)\cdot E\right]$$

$$E = \frac{C\cdot q_0}{\mu_0(\mu_0-q_0)}$$

$$h = \mu - \mu_0 + q_0$$

$$y = 1 - \frac{h}{q}$$

μ = capacity of the intersection entry during the peak period of duration T
μ_0 = capacity of the intersection entry after the peak period
q = minor street volume during the peak period of duration T
q_0 = minor street volume after the peak period (all in veh/h)
T = duration of the peak period (h)

C is again similar to the factor C mentioned for the M/G/1 system, where:

C = 1 for unsignalized intersections; and
C = 0.5 for signalized intersections.
(Refer to Kimber and Hollis, 1979)

This delay formula has proven to be quite useful to estimate delays and it has a quite reliable background particularly for temporarily oversaturated conditions. It can be strongly recommended, especially for the use in computer programs.

In the original version of the formula (Kimber,Hollis, 1979) instead of C, the term k * C, where k = 2, has been used to represent the effects of a sharp peak within the period of maximum volumes (for more details see: Kimber, Hollis, 1978). This extension, however, has not turned out to produce reliable results for longer peak periods, like e.g. T= 1 h.

Conclusion to queueing theory

The previous chapter has indicated some results from queueing theory which provide solutions to the performance of unsignalized intersections operating with gap acceptance behavior. The discussed solutions, however, only tried to deal with the simple 2-stream case of Figure 1. Although the description is clear, the queueing theory solutions of the gap acceptance problems are complicated. They do not fulfill the needs of traffic engineers in practice. These are:

- Solutions for arbitrary time-dependencies;
- Solutions for distributed t_c, t_f values;
- Solutions for non-poisson traffic streams; and
- Solutions for the hierarchical system mentioned on page 11.

Of course, practical solutions should solve each of these 4 problems simultaneously. Finally, one must state that a general analytical solution has not yet been found.

STOCHASTIC SIMULATION

Due to these reasons analytical methods are not capable of providing a completely satisfying practical solution. The modern tool of stochastic simulation, however, is able to overcome all the problems very easily. The degree of reality of the model can be increased to any desired level. It is only restricted by the efforts one is willing to undertake and by the available (and tolerable) computer time. Therefore, stochastic simulation models for unsignalized intersections were developed very early (Steierwald, 1961; Boehm, 1968). More recent solutions are reported from U.K. (Salter 1982), from Germany (Zhang 1988; Grossmann 1988, 1992) from Canada (Chan, Teply, 1991) and from Poland (Tracz, 1991).

Speaking about stochastic simulation, we have to distinguish between 2 levels of complexity:

- *Point process models*: Here cars are treated like points, i.e. the length is neglected. As well, there is only limited use of deceleration and acceleration. Cars are regarded as if they were "stored" at the stop line. From here they depart according to the gap acceptance mechanism. The effect of limited acceleration and deceleration can, of course, be taken into account using average vehicle performance values (cf. Grossmann, 1988). The advantage of this type of simulation model is the rather small computer time needed to run the model for realistic applications. One such model is KNOSIMO (Grossmann, 1988, 1991). It is capable of being operated by the traffic engineer on his/her personal computer during the process of intersection design. KNOSIMO in its present concept is based on German conditions. One of the conditions is the restriction to single-lane traffic flow for each direction of the main street. Chan, Teply (1991) found some easy modifications to adjust the model to Canadian conditions as well. Moreover, the source code of the model could easily be adjusted to traffic conditions and driver behavior in other countries.
- *Car tracing models*: These models take into account the space which cars occupy on a road together with the car-following process in great detail. An example of this type of model is described by Zhang (1988). This type of model requires lengthy computer processing times.

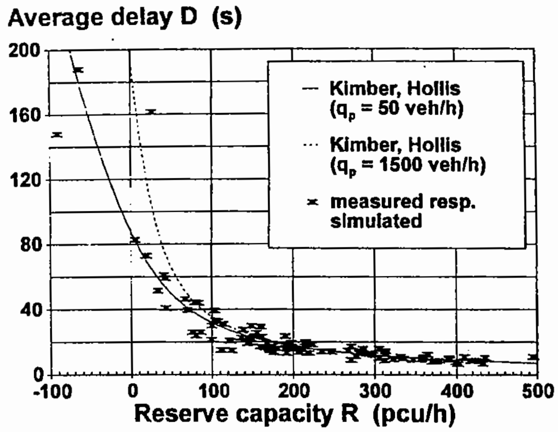


FIGURE 6 Average delay D in relation to reserve capacity R.

Both types of models are useful for research purposes. The models can be used to develop relationships which can then be represented by regression lines or other empirical evaluation techniques. However, only the first model type — up to now — is fast enough to be used directly in practice.

THE IMPORTANCE OF RESERVE CAPACITY

Independent of the model used to estimate average delays, the reserve capacity (R) plays an important role

$$R = q_{e,\max} - q_n \quad \text{(veh/h)} \tag{2.7.1}$$

In the HCM 85 it is used as the measure of effectiveness. This is based on the fact that average delays are closely linked to reserve capacity (cf. eq. 2.5.5). This close relationship is shown in Figure 6. Based on this relationship, a good approximation of the average delay can also be expressed by reserve capacities. What we also see is that – as a practical guide – a reserve capacity:

$$R > 100 \text{ veh/h}$$

generally ensures an average delay below 45 s.

CAPACITY FOR STREAMS OF RANK 3 AND RANK 4

No rigorous analytical solution is known for the derivation of the capacity of rank-3 movements such as the left-turner from the minor street at a T-junction (movement 4 in Figure 4, right side). Here, the gap acceptance theory uses the impedance factors p_0 as an approximation. p_0 for each movement is the probability that no vehicle is queueing at the entry. This is given with sufficient accuracy by eq. 2.5.9 or better with the two service time equation 2.5.11. Only during the part $p_{0,\,\text{rank 2}}$ of the total time, vehicles of rank 3 can enter the intersection due to highway code regulations.

Therefore, for rank-3 movements, the basic value $q_{e,max}$ for the potential capacity must be reduced to $p_0 \cdot q_{e,max}$ to get the real potential capacity q_e.

$$q_{e,rank3} = p_{0,rank2} \cdot q_{e,\max,rank3} \tag{2.8.1}$$

For a T-junction, this means

$$q_{e,4} = p_{0,7} \cdot q_{e4,\max}$$

For a cross-junction, this means

$$q_{e,5} = p_x \cdot q_{e5,\max} \tag{2.8.2}$$

$$q_{e,11} = p_x \cdot q_{e11,\max} \tag{2.8.3}$$

with

$$p_x = p_{0,1} \cdot p_{0,7}$$

Here the index numbers refer to the index of the movements according to Figure 4. The values of $p_{0,1}$, $p_{0,5}$, $p_{0,7}$, and $p_{0,11}$ can be calculated according to eq. 2.5.9.

For rank-4 movements (left turners at a cross-intersection), the dependancy between the p_0 values in rank-2 and rank-3 movements must be considered. This has been evaluated by a long series of simulation runs by Grossmann (1991; cf. Brilon, Grossmann, 1991b).

Figure 7 shows the statistical dependance between queues in streams of ranks 2 and 3.

In order to calculate the maximum capacity for the rank-4 movements (no.4 and 10), the auxiliary factors $p_{z,5}$ and $p_{z,11}$ should be calculated first:

$$p_{z,i} = f(p_{y,i}) \tag{2.8.4}$$

$$p_{z,i} = 0.65 \cdot p_{y,i} - \frac{p_{y,i}}{p_{y,i}+3} + 0.6 \cdot \sqrt{p_{y,i}}$$

(this equation is represented by Figure 7)
where:

TABLE 2 EVALUATION OF CONFLICTING PRIORITY VOLUME q_p (THE INDICES REFER TO THE TRAFFIC STREAMS DENOTED IN FIGURE 4)

Subjective Movement	No.	Conflicting Traffic Volume q_p
Left Turn from Major Road	1	$q_8 + q_9{}^3$
	7	$q_2 + q_3{}^3$
Right Turn from Minor Road	6	$q_2{}^2 + 0.5q_3{}^1$
	12	$q_8{}^2 + 0.5q_9{}^1$
Through Movement from Minor Road	5	$q_2 + 0.5q_3{}^1 + q_8 + q_9{}^3 + q_1 + q_7$
	11	$q_2 + q_3{}^3 + q_8 + 0.5q_9{}^1 + q_1 + q_7$
Left Turn from Minor Road	4	$q_2 + 05q_3{}^1 + q_8 + q_1 + q_7 + q_{12}{}^{4,5,6} + q_{11}{}^5$
	10	$q_8 + 0.5q_9{}^1 + q_2 + q_1 + q_7 + q_6{}^{4,5,6} + q_5{}^5$

Notes:

[1] If there is a right-turn lane, q_3 or q_9 should not be considered.
[2] If there is more than one lane on the priority road, q_2 and q_8 include only the volume in the right lane.
[3] If right-turning traffic from the major road is separated by a triangular island and has to comply with a YIELD- or STOP-sign, q_9 and q_3 need not be considered.
[4] If right-turning traffic from the minor road is separated by a triangular island and has to comply with a YIELD- or STOP-sign, q_6 and q_{12} need not be considered.
[5] If movements 11 and 12 are controlled by a STOP-sign, q_{11} and q_{12} should only be considered by half of their values. If movements 5 and 6 are controlled by a STOP-sign, q_5 and q_6 should only be considered by half of their values.
[6] It can also be justified to omit q_6 or q_{12} or to reduce it to half of its value if the minor approach area is very wide.

i = 5, 11
= index no. of the movements according to Figure 4

$p_{y,i}$ = $p_x \cdot p_{0,i}$

f = functional relation according to Figure 7 (the product $p_{y,i}$ is used as the entry value [horinzontal axis] into the diagram)

Then, the maximum capacity of the left-turn movements from the minor road at the cross-intersection is calculated as:

$$q_{e4} = p_{z,11} \cdot p_{0,12} \cdot q_{e4,max} \qquad (2.8.5)$$

$$q_{e10} = p_{z,5} \cdot p_{0,6} \cdot q_{e10,max} \qquad (2.8.6)$$

If the right-turn movements from the minor road (stream no. 6 or 12) are separated by a triangular island and have to comply with a YIELD or STOP sign, $p_{0,6}$ or $p_{0,12}$ must not be included in eq. 2.8.5 and 2.8.6 (cf. table 2, note 4).

In each case, the maximum potential capacity $q_{e,max}$ should be calculated according to paragraph 'capacity' (page 11) using the sum of all conflicting traffic volumes with higher priority than the priority of the traffic stream in question. To enable the correct additions, table 2 can be used. This table, basically, corresponds to the German guidelines both from 1972 and 1991 as well as to the HCM 1985.

Several authors have objected that traffic streams of rank 2 and 3 are introduced twice into the calculations

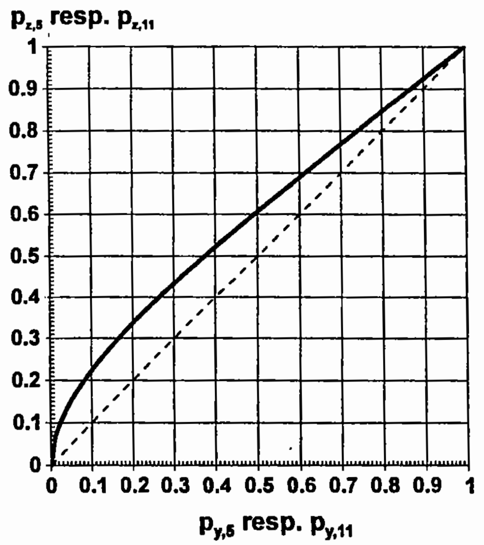


FIGURE 7 Reduction factor to account for the statistical dependence between streams of rank 2 and 3.

according to this procedure. These streams are, on the one hand, added to the main street volumes (cf. table 2). On the other hand, they are introduced into the impedance factors p_0 by which the basic capacities $q_{e,max}$ of rank-3 and rank-4 streams are diminished to calculate the actual capacities q_e. Brilon (1988b, cf. Figure 7 and 8) has discussed arguments which support this double introduction.

The reasons for this are as follows:

1. During times of queueing in rank-2 streams (e.g. left turners from the major street), the rank-3 vehicles (e.g. left turners from the minor street at a T-junction) should not enter the intersection due to traffic regulations and the highway code. Since the portion of time provided for rank-3 vehicles is p_0, the basic capacity calculated from eq. 1 for rank-3 streams has to be diminished by the factor p_0 for the corresponding rank-2 streams This effect is taken into account by the impedance factors according to eq. 2.8.1 - 2.8.6.

2. Even if no rank-2 vehicle is queueing, these vehicles influence rank-3 operations, since a rank-2 vehicle approaching the intersection within a time of less than t_c prevents a rank-3 vehicle from entering the intersection. The inclusion of rank-2 and rank-3 movement flows into the equations of table 2 takes care of this effect.

A recent research project (Grossmann, 1991) has proven that among the possibilities considered, the described approach is the best one to account for these relations.

Note 5 in table 2 is related to the fact that an approaching opposite vehicle which is forced to slow down due to the stop sign does not impede vehicles of movement 4 to enter the intersection (i.e. no or less effect of type b). The influence of stopped vehicles in movements 11 and 12 is taken into account by equations 2.8.1 - 2.8.6

Note 6 in table 2 is related to the fact that, at wide intersections, movement 6 does not necessarily impede vehicles of movement 4.

SHARED LANE FORMULA

Shared Lanes on the Minor Street

If more than one minor street movement is operating on the same lane, the so-called "shared lane equation" can be applied. It calculates the total capacity q_m of the shared lane, if the capacities of the corresponding movements are known (derivation e.g. Harders, 1968).

$$\frac{1}{q_m} = \sum_{i=1}^{m} \frac{b_i}{q_{ei,max}} \qquad (2.9.1)$$

q_m = capacity of the shared lane (veh/h)
$q_{ei,max}$ = capacity of movement i if it would operate on a separate lane (veh/h)
b_i = proportion of volume of movement i of the total volume on the shared lane
m = no. of movements on the shared lane.

The formula is also used by the HCM (1985, eq. 10-1).

This equation is of general validity regardless of the formula for the estimation of $q_{e,max}$ and regardless of the rank of priority of the three traffic movements. The formula can also be used if the overall capacity of one traffic stream should be calculated, if this stream is formed by several partial streams with different capacities, e.g. by passenger cars and trucks with different critical gaps.

Shared Lanes on the Major Street

In the case of a single lane on the major street shared by right-turning and through movements (movements no. 2 and 3 or 8 and 9 in Figure 4), one can refer to table 2, where this case is addressed in note 1.

If left turns from the major street (movements no. 1 and 7 in Figure 4) have no separate turning lanes, vehicles in the rank-1 movements no. 2 and 3, and 8 and 9 respectively in Figure 4 may also be obstructed by queueing vehicles of those streams. The factors $p_{0,1}^*$ and $p_{0,7}^*$ indicate the probability that there will be no queue in the respective shared lane. They might serve for a rough estimate of the disturbance that can be expected and can be approximated as follows (Harders, 1968):

$$p_{0,i}{*} = 1 - \frac{1 - p_{0,i}}{1 - \frac{q_j \cdot t_{Bj} + q_k \cdot t_{Bk}}{3600}} \qquad (2.9.2)$$

where:

i	=	1 resp. 7	(cf. Figure 4)
j	=	2 resp. 8	
k	=	3 resp. 9	
q_j	=	volume of stream j	(veh/h)
q_k	=	volume of stream k	(veh/h)
t_B	=	follow-on time required by a vehicle in stream j or k ($1.7\ s < t_B < 2.5\ s$, e.g. $t_B = 2\ s$)	(s)
$p_{o,i}$	=	impedance factor for movement i (eq. 2.5.9)	(-)

In order to account for the influence of the queues in the major street approach lanes on the minor street streams no. 4, 5, 10, and 11, the values $p_{0,1}$ and $p_{0,7}$ according to eq. 2.5.9 have to be replaced by the values $p_{0,1}^*$ and $p_{0,7}^*$ according to eq. 2.9.2. This replacement should be made from eq. 2.8.1 to 2.8.4.

Conclusion to Gap Acceptance Theory

The gap acceptance model tries to model driver behavior on a microscopic level. Every single driver is modelled by his behavior which is represented by the critical gap t_c and the follow-on time t_f. Based on this microscopic model, the estimates of capacity, delay and queue length can be achieved by:

- Analytical queueing theory; and
- Stochastic simulation.

Analytical queueing theory is able to contribute to the understanding of many interrelations between variables. It is, however, quite difficult to obtain realistic results that represent every detail. Nevertheless, many rough but quite reliable estimation procedures being used in practice in several countries are based on queueing theory solutions.

Stochastic simulation as a solution of the gap acceptance problem is able to find more detailed and more realistic results. The only drawback, at present, is that the user must have a computer available.

The guideline procedures which are, up to now, still paper-and-pencil methods, have been based on both types of solutions for the gap acceptance model.

One drawback of the gap acceptance models, however, is inherent to both solutions: The gap acceptance mechanism is also assumed under conditions where, in reality, other mechanisms prevail. This encompasses particularly:

- Drivers from the minor street forcing their way into the major stream (gap forcing).
- Reversed priority where the major stream drivers do not accept priority.
- Capacities restricted by traffic congestion on the main street.

These types of behavior can be observed especially under heavy traffic concentrations.

EMPIRICAL REGRESSION THEORY

The capacity of the simple 2-stream problem can also be evaluated based on an empirical analysis without any queueing theory. This is the type of solution used in the United Kingdom.

The fundamental idea of this solution is as follows: Again, we look at the simple intersection (Figure 1) with one priority traffic stream and one nonpriority traffic stream during times of a steady queue (i.e. at least one vehicle queueing on the minor street). During these times, the volume of traffic departing from the stop line is the capacity. This capacity should depend on the priority traffic volume q_p during the same time period. To derive this relationship, observations of traffic operations of the intersection have to be made during periods of oversaturation of the intersection. The total time of observation then is divided into periods of constant duration, e.g. 1 minute. During these 1-minute intervals, the number of vehicles in both the priority flow and the entering minor street traffic have to be counted. These counts have to be transformed (e.g. for 1-minute intervals by multiplication by 60) into traffic volumes with the unit of veh/h. Normally, these data points are scattered over a wide range. Then, these data points are represented by a linear regression line (cf. Figure 8). On average, half of the variation of data points results from the evaluation technique in 1-minute intervals. Only the second half is created by variations in driver behavior or geometrical design of intersections. Under practical conditions, evaluation intervals of more than 1 minute (e.g. 5 minutes) cannot be used, since this normally leads to an unacceptable reduction of the sample size because there are only few oversaturation periods of such length at normal unsignalized intersections.

As a result, the method would produce linear relations for $q_{e,max}$:

$$q_{e,max} = b - c \cdot q_p \tag{3.1}$$

Instead of a linear function, other types of regression could be used as well, e.g.

$$q_{e,max} = A \cdot e^{-Bx} \tag{3.2}$$

Here, the regression parameters A and B could be evaluated out of the data points by linear regression after logarithmic transformation of eq. 3.2. This type of equation is of the same form as Siegloch's capacity formula (eq. 2.4.2). This analogy shows that $A = 3600/t_f$.

If the empirical regression theory is applied to the complete hierarchy of traffic movements at an intersection (cf. Figure 4), eq. 3.1 can be amplified into a multinominal linear-regression equation of the type

$$q_{x,max} = d - \sum_i g_i \cdot q_{p,i} \tag{3.3}$$

= potential capacity for traffic movement x (veh/h)

with

- d = constant
- x = index for nonpriority traffic movements (no. 1, 4, 5, 6, 7, 10, 11, 12)
- i = index for movements with priority over movements x
- g_i = constant for movement i
- $q_{p,i}$ = traffic volume for movement i (veh/h)

The constants d and g_i are evaluated by multiple linear regression from the data points. This is the type of formula that is used in the British approach (cf. page 27). In general, some interrelations between traffic movements of different ranks of priority should be taken into account as well. Therefore, the regression equation should normally be of the type

$$q_{x,max} = h - \sum_i k_i \cdot q_{p,i} - \sum_i \sum_j k_{ij} \cdot q_{pi} \cdot q_{pj} \quad \text{(veh/h)} \tag{3.4}$$

with

- h = constant
- k_i = constant for movement i
- x,i = as in eq. 3.3
- j = index for movements with priority over movement x, with
  - a) $i \neq j$
  - b) rank (i) $\neq$ rank (j)

The sums have to be formed over each possible combination of i and j according to the regarded movement x. The 3rd term with the product of priority volumes intends to take account of the impedance effects between movements of different ranks of priority.

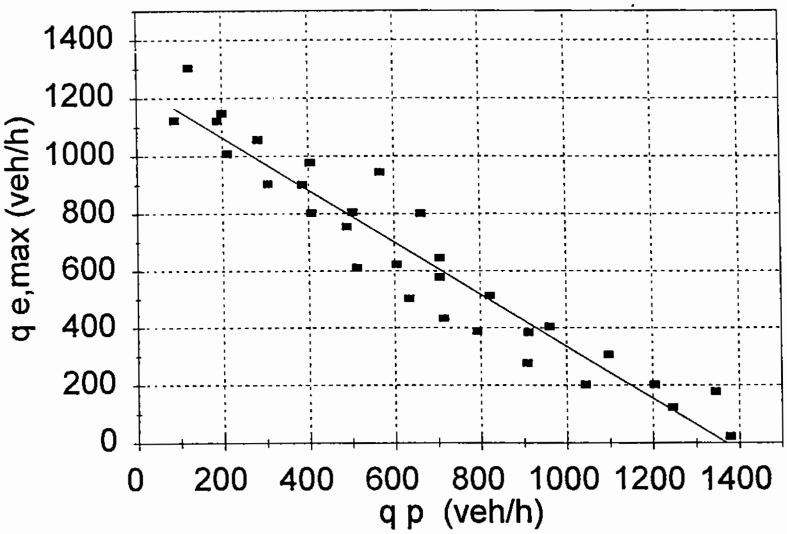


FIGURE 8 General outline for the use of linear regression theory.

In case of a T-junction, eq. 3.4 would be written (cf. Figure 4, right side):

$$q_4 = h - k_2 \cdot q_2 - k_3 \cdot q_3 - k_7 \cdot q_7 - k_8 \cdot q_8 \\ - k_{27} \cdot q_2 \cdot q_7 - k_{37} \cdot q_3 \cdot q_7$$

Although this general eq. (3.4) would be an adequate solution, no report on investigations of such comprehensive regression analysis is known.

In addition to the influence of priority stream traffic volumes on the minor street capacity, the influence of geometric layout of the intersection can be investigated. To do this, the constant values

b and c	eq. 3.1
d and g_i	eq. 3.3
or h, k_i and k_{ij}	eq. 3.4

can be related to road widths or visibility or even other characteristic values of the intersection layout by another set of linear regression analysis.

The estimation of delays and queue lengths is again derived using queueing theory. Here, however, these equations use the maximum entry flow as an input and it is not calculated from t_c and t_f values. In practice, these empirical regression equations are always combined with the Kimber-Hollis theory (eq. 2.5.17).

Here are some of the pros and cons of the empirical regression technique for the investigation of unsignalized intersection capacity:

The advantages are:

- There is no need to establish a theoretical model.
- Reported empirical capacities are used.
- Influence of geometric design can be taken into account.
- Effects of priority reversal and forced priority are taken into account automatically.
- There is no need to describe driver behavior in detail.

The disadvantages are:

- Transferability to other countries or other times (driver behavior might change over time) is quite limited: For application under different conditions, a very big sample size must always be evaluated.
- No real understanding of traffic operations at the intersection is achieved by the user.
- The equations for four-legged intersections are too complicated.
- The derivations are based on driver behavior under oversaturated conditions.
- Each situation to be described with the capacity formulae must be observed in reality. On one hand, this requires a large effort for data collection. On the other hand, many of the desired situations are found infrequently, since these oversaturated intersections have been often already changed into signalized intersections or roundabouts or have been modified by other measures.

KYTE'S METHOD

Kyte (1989; and Kyte et al., 1991) proposed another method for the direct estimation of unsignalized intersection capacity. The idea is based on the fact that the capacity of a single-channel queueing system is the inverse of the average service time. The service time W at the unsignalized intersection is the time which a vehicle spends in the first position of the queue. Therefore, only the average of these times W has to be evaluated by observations to get the capacity.

Under oversaturated conditions with a steady queue on the minor street approach, each individual value of this time in the first position can easily be observed as the time between two consecutive vehicles crossing the stop line. In this case, however, the observations and analyses are equivalent to the empirical regression technique (chapter 3).

Assuming undersaturated conditions, however, the time each of the minor street vehicles spends in the first position could be measured as well. Again, the inverse of the average of these times is the capacity. Examples of measured results are given by Kyte et al. (1991).

From a theoretical point of view, this method is correct. The problems turn out if measurements (e.g. by video taping) are to be processed. Here it is quite difficult to define the beginning and the termination of the time spent in the first position in a consistent way. If this problem is solved, this method provides an easy procedure for estimating the capacity for a movement from the minor street even if this traffic stream is not operating at capacity.

The further evaluation of these measurement results corresponds to the methods of chapter 3. Again, regression techniques can be employed to relate the capacity estimates to the traffic volumes in those movements with a higher rank of priority.

EVALUATION OF THE PRACTICES OF DIFFERENT COUNTRIES

The approach taken in this report has been to describe the traffic theory that is applicable to unsignalized intersections. Some of this theory will not be available in currrent practices, but a measure of the sophistication of the technique is the extent to which they use aspects of existing theory.

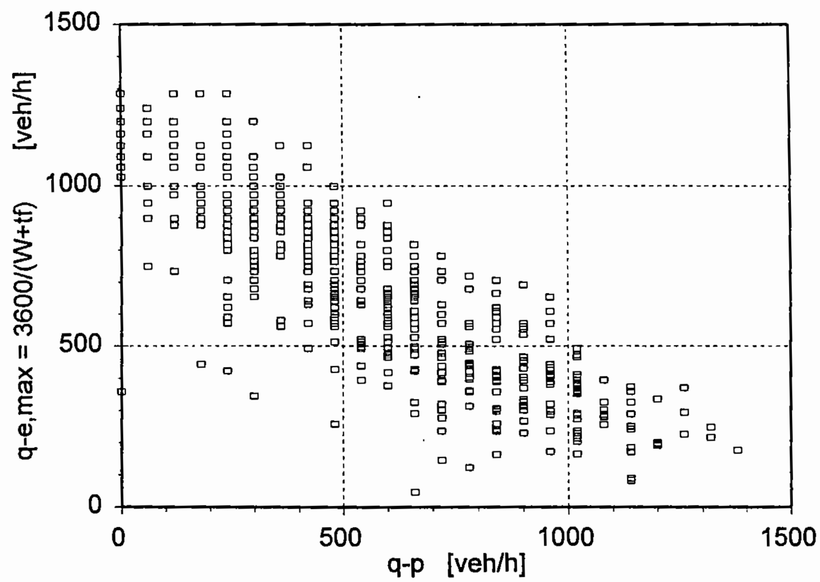


FIGURE 9 Example for Kyte's method: Relationship between the capacity estimated by the service time W in the first position of the queue plus the follow-on time t_f (here t_f = 2.5 s) . This example was evaluated to estimate the capacity of a single-lane entrance to a roundabout based on measurements at 5 different sites in Germany (Brilon, Stuwe, Drews, 1993).

AUSTRALIA

AUSTROADS (1986) TECHNIQUE

The technique in the AustRoads Guide to Traffic Engineering Practice (AustRoads, 1986) is their current recommended technique for analyzing unsignalized intersections. The critical gap and the follow-on times have been estimated for a large number of possible geometries. The list of critical acceptance gaps and follow-on time values are shown in Table 3.

The opposing flow is considered to be random and consequently to have a total opposing flow equal to the sum of the individual opposing flows. The total absorption capacity is then estimated for say a priority 3 stream to enter a priority 1 stream. To estimate the impedance of a priority 2 stream, the guide recommends that the priority 3 stream absorption capacity be reduced by the total flow of priority 2 stream vehicles.

Delay is estimated using Tanner's (1962) relationship and with a minimum headway of 0 s. This is the same as assuming that all stream arrivals are random.

This technique lacks a reasonable impedance model, time dependency, and the influence of the geometric parameters.

TABLE 3: t_c AND t_f VALUES FOR AUSTRALIAN CONDITIONS

	Critical Acceptance Gap t_c (s)	Follow-up Headway t_f (s)
Crossing Maneuvers		
Two-directional stream		
2-lanes	5	3
4-lanes	8	5
One-directional stream		
2-lanes	4	2
3-lanes	6	3
4-lanes	8	4
Merging Maneuvers		
Merging, e.g. from acceleration lane	3	2
Right Turning Maneuvers		
Across single lane flow		
Good turning conditions	4	2
Difficult turning conditions	5	3
Across 2-lane flow	5	3
Across 3-lane flow	6	4

Note: The listed values for t_c and t_f assume good sight distance conditions and reasonable grades. Allowance should be made for extraordinary conditions. Values of critical gap (and follow-up headway) required for left turn maneuvers become very large (t_a = 14 to 40 s) if it is assumed that through traffic is not impeded. Most drivers would not wait for such gaps and would enter on gaps of 5 to 8s, thus requiring the through vehicle to slow down or change lanes. Judgment needs to be exercised in the selection of the appropriate critical acceptance gap for left turn maneuvers to suit the circumstances.

BENNETT'S METHOD

Bennett's method (Bennett, 1984) is a well regarded technique that has not yet been accepted as a national standard. It is very similar to the German approach (Grossmann, 1991) but it was not as rigorously tested. Nevertheless, it is worth explaining. It is essentially a mixture of gap acceptance and queueing theory. It does however produce reasonable solutions which are better than those given in AustRoads (1986).

Bennett stated that a rank-3 stream vehicle would not depart unless:

- There is an adequate gap in rank-1 stream;
- There is an adequate gap in rank-2 stream; and
- There is no queue of rank-2 stream vehicles waiting for an acceptable gap in rank-1 stream.

Bennett calculated the probability for these three conditions and produced the notion of an equivalent flow, q_a and p_a, which could be used in the following equation developed from eq. 2.4.2 to estimate capacity:

$$q_{e3,max} = \frac{q_a \cdot e^{-p_a t_c}}{1 - e^{-p_a t_f}}$$

with

$$p_a = p_1 + p_2 - \ln\left(1 - \frac{q_2}{q_{e2,max}}\right) \qquad \text{(veh/s)}$$

$q_a = p_a \cdot 3600$ (veh/h)
p_1 = traffic flow in rank-1 stream (veh/s)
p_2 = traffic flow in rank-2 stream (veh/s)
q_2 = traffic flow in rank-2 stream (veh/h)

Theoretically, this equation gives the correct answers when $q_2 = 0$ or $q_2 = q_{e2,max}$, but it substantially underestimates $q_{e3,max}$ as q_1 approaches zero.

Bennett recommends that the designer use the AustRoads (1986) values of the critical gap and follow-on time.

Bennett's method provides a reasonable practical method of estimating impedance using the AustRoads (1986) gap acceptance values. It should be extended to give time dependent estimates.

THE FORMER CZECHOSLOVAKIA

The practice in The Former Czechoslovakia is described by Jirava, Karlicky (1988) and Medelska (1992). The Former Czechoslovak standard ON 73 61 02 for the design of all types of junctions, the method according to Harders (1968), which is also a basis of the HCM 1985, is accepted as the concept for capacity calculations.

Values for critical gaps t_c have been evaluated. They vary over a range from 5.0 to 8.5 s for passenger cars and from 6.0 to 10.5 s for trucks depending on the type of movement and on the speed level on the major street.

More recent validation data gave smaller values for t_c between 3.0 and 8.0 s. The values for the follow-on time varied between 2.5 and 3.9 s. These times depend on the type of movement, the corresponding t_c value, percentage of heavy vehicles and the position of the vehicle in the queue.

At present, more research activities are directed into the suitable type of the major stream headway distribution.

GERMANY

A German guideline published in 1972 was the basis of chapter 10 of the HCM 1985. Here Harders' formula for capacity (eq. 2.4.1) was applied. As the only measure of effectiveness, the reserve capacity was used. It became evident that the procedure had a couple of drawbacks so that it ceased to be used in Germany by the late 1970s and the early 1980s.

The German Road Research Association (FGSV, 1991) has recently published a new procedure for TWSC intersections. The whole concept has been developed by Brilon, Grossmann (1991). It is based on gap acceptance theory. Critical gap t_c and follow-on times t_f were obtained from Harders (1976), although some doubts have been raised regarding the validity of this data base. These values are very strongly related to average velocities on the major street. They are, however, independent of the degree of saturation. A new research project to reinvestigate the t_c and t_f values should be started in the near future. The basic potential capacity is now evaluated with Siegloch's formula (eq. 2.4.2). It has been shown that:

- Non-constant t_c and t_f values (variation among different drivers) decreases capacity; and
- Bunching in traffic increases capacity, compared with the poisson-assumptions which are the basis of Siegloch's formula.

It happens, however, that in most cases these two effects cancel each other. Therefore, also under realistic conditions, Siegloch's formula is still a good approximation for the basic capacity formula. The impedance effects due to the hierarchy of streams have been taken into account based on simulations performed with KNOSIMO. One essential point which is an important difference compared to the former procedure (e.g. chapter 10 in HCM 1985) is the use of eq. 2.5.9. Another new feature is the sequence of formulas 2.8.1 until 2.8.6 of this paper including the correction of the impedance factors for rank-4 movements according to Figure 7.

In addition, the new German guideline also contains a procedure for the rough estimation of delays. Comments in English to the new procedure are given by Brilon, Grossmann, Stuwe (1991). The new procedure in its fundamental concept is very similar to the old German guideline of 1972 and, thus, to chapter 10 of the HCM 85. Only the problems with these former standards are avoided by the new method. Besides the guideline procedure, the simulation program KNOSIMO (Brilon, Grossmann, 1989; Grossmann, 1988) is used in practice in Germany as a standard method as well.

The new method is universal in such a way that only the t_c and t_f values for every individual country have to be evaluated. Such a data base facilitates an easy adaptation of the method.

GREAT BRITAIN/UNITED KINGDOM

Practice of capacity estimations at unsignalized intersections in the United Kingdom is concentrated on the empirical regression approach (cf. chapter 3). Based on a tremendous amount of measurement data, Kimber and Coombe (1980) published a set of regression functions to predict capacities. These equations are restricted to T-junctions, since unsignalized cross-junctions normally are avoided in modern British street design or they are replaced by a roundabout.

Using the British notation (cf. Figure 10), these regression functions are applied in the United Kingdom:

$$\mu_{B-C} = Z_B[745 - Y(0.364q_{A-C} + 0.144q_{A-B})] \quad \text{pcu/h} \tag{6.1}$$

where

$$Z_B = (1+0.094(w_{B-C} - 3.65))(1+0.0009(V_{rB-C} - 120))$$

and

$$Y = (1-0.0345W)$$

$$\mu_{B-A} = X_B[627 + 14W_{CR} - Y(0.364q_{A-C} + 0.144q_{A-B} + 0.229q_{C-A} + 0.52q_{C-B})] \quad \text{pcu/h} \tag{6.2}$$

where

$$X_B = (1+0.094(w_{B-A}-3.65)(1+0.0009(V_{rB-A}-120))(1+0.0006V_l - 150))$$

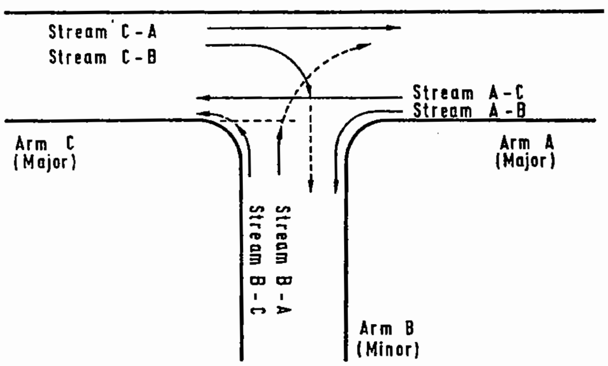


FIGURE 10 Traffic streams at a 3-arm major/minor junction.

$$\mu_{C-B} = X_c[745 - 0.364Y(q_{A-C} + q_{A-B})] \quad \text{pcu/h} \qquad (6.3)$$

where

$$X_c = (1+0.094(w_{C-B} - 3.65))(1+0.0009(V_{rB-C} - 120))$$

Here, μ stands for the potential capacity (denoted as $q_{e,max}$ elsewhere in this paper). q is the existing traffic volume. w is the width of the roadway available for nonpriority movements. W is the total width of the major street. W_{CR} is the width of the central reserve. V is the visibility distance from the minor street to the vehicles on the major street. Indices r and l stand for visibility to the right and to the left. For more details, see Kimber and Coombe (1980).

Thus, the determinants of the minor street capacity are:

- Lane widths of the minor street and lane configuration;
- Width of the major street;
- Dual carriageway sites: width of the central reserve; and
- Visibility distances.

On the other hand, the basic investigations showed no influence of the

- Velocities of the major street traffic, and
- Approach gradients between ± 8% on the minor street capacities.

For practical purposes, the computer program PICADY is widely used in the United Kingdom. It can be used for T- and cross-junctions (crossroads and staggered junctions) with a range of configurations of the major and minor streets. As capacity equations for T-junctions, eq. 6.1 - 6.3 are used. These are extended into equations for cross-junctions by Semmens (1985). Demand flows, turning proportions and vehicle type information have to be entered by the user. This results in estimations of capacity, delays and queue lengths which take into account

- The time dependancy of traffic volumes; and
- The effects of temporary oversaturation.

Due to the program's nature, it cannot be adjusted by the user to take into account external conditions and driver behavior which might be different from the British experience.

JAPAN

The Japanese technique, as documented by the Japan Society of Traffic Engineers (1988), uses exponential traffic with only little evidence of the appropriate values of the critical gap and the follow-on time. They have used the shared lane approach of Harders and the US HCM (1985).

The technique does describe the effects of the impedance of the intermediate priority streams in the text but gives only brief details on how to apply the concepts.

The technique lacks a full description of the impedance effects, the influences on the gap acceptance parameters, and the influence of the geometry.

NETHERLANDS

In the Netherlands, a standardized procedure for the calculation of unsignalized intersections has not been implemented so far (Middelham, 1992). Several methods have been (or are still) used throughout the country, like the Swedish and the German or the HCM (1985) methods as well as the SIDRA (Akcelik, 1991).

A simulation program called FLEXSYT (Middelham, 1986) is used for traffic management studies by simulating traffic on a microscopic scale. Unsignalized intersections can be modelled.

NEW ZEALAND

TRANSIT, NEW ZEALAND METHOD

Transit New Zealand (standard-setting organization for New Zealand Roads) has copied the AustRoads (1986) method from Australia without change. Consequently, the comments applied to AustRoads (chapter 5) also apply to the recommended New Zealand technique.

FISK METHOD

Although Fisk's (1989) technique is not the accepted New Zealand practice, it is however well regarded in NZ. Her methods are based on the notion that drivers have different critical gaps when crossing or moving across different lanes. For instance if a driver is crossing a three lane road from a minor road to make a left hand turn into the conflicting traffic stream on the other side of a median, then the driver might have a low critical gap for crossing the lane with slow moving vehicles, a larger critical gap when crossing the lanes with faster moving vehicles and a lower critical gap when merging into the conflicting traffic on the other side of the median. The crossing process is a complex one and the theory simplifies the approach. It is difficult to appreciate whether this extra complication of the theory is justified, especially since there seems to be little or no empirical evidence that drivers do in fact have different critical gaps when crossing each stream of a number of streams. This approach does not account for the impedance of streams with a range of priority levels as shown in Figure 4.

Fisk's equation is:

$$Q_e = q_1 \frac{\prod \left[(1-t_m p_{1i}) e^{-p_1 t_{ci}}\right]}{e^{-p_1 t_m}(1-e^{-p_1 t_f})} \tag{7.1}$$

where

t_f	=	follow-on time	(s)
t_{ci}	=	critical gap that is required for a driver to cross major stream lane i	(s)
p_{1i}	=	flow in major stream lane i	(veh/s)
p_1	=	major stream flow ($p_1 = \Sigma p_{1i}$)	(veh/s)
q_1	=	major stream flow	(veh/h)

This equation is very similar to Tanner's (1967) equation and Troutbeck's (1986) equation which gives the entry capacity when there are a number of streams, each with different degrees of bunching. This latter equation is:

$$Q_e = \lambda' \frac{\prod \left[(1-t_m q_{1i}) e^{-\lambda' T}\right]}{e^{-\lambda' t_m}\left(1-e^{-\lambda' t_f}\right)} \tag{7.2}$$

where λ' is the sum of λ_i and λ_i is given by equation (2.2.4) above. For Tanner's (1962) equation $\alpha_i = (1 - t_m \cdot q_i)$ and $\lambda_i = q_i$. The equation by Tanner (1967) would seem to be more useful.

Fisk and Tan (1989) have documented an approach for a delay analysis for priority intersections. Their approach uses the Catling (1977) transformation from the steady state to the deterministic equations based on two M/M/1 queueing systems for two different customer types as in the shared lane case. It should be noted that this transformation is only an empirical technique and it is not based on any theoretical basis.

The equation of Fisk and Tan provides a useful solution to the shared lane problem given exponential minor stream traffic and random service times. Fisk and Tan's equations do not approach the correct solutions for an intersection with single lane roads as the degree of saturation (and hence the flow) in one lane approaches zero. Their equations are also not able to predict the outcome when both lanes have the same service rate.

Fisk's results are based on a mixture of gap acceptance and queueing theory. The advantages of this approach do not warrant its use over Tanner's models and it does not fully account for the effects of bunching.

POLAND

The Polish method is based on gap acceptance. It uses critical gaps, move-up times and minimum headways in the main stream. The evaluation of the method, however, is not performed by analytical methods. Instead it is based on the simulation model of traffic operations including the flow demand submodel with non-stationarity and bunching of traffic flow as well as randomness of the gap acceptance process. The model has been calibrated by measurements in Poland and by some foreign results. The structure of the calculation procedure is similar to that of the HCM 85.

Two sets of potential capacity graphs for the major road left turns and the minor road movements were derived from simulation.

The potential capacities are adjusted for the effects of:

- Approach lane width and pedestrian activity;
- Visibility;
- Impedance;
- Grade and percentage of heavy vehicles; and
- Shared lanes.

The impedance graphs take into account lane configurations of the impeding movements, including provision of separate lanes.

The performance measures include:

- Qualitative LOS classification (4 levels); and
- Quantitative measures: delay and stops, degree of saturation.

Additional material can be found in Tracz (1991), Tracz, Chodur and Gondek (1990), Tracz (1988), Chodur and Gaca (1988).

The procedure includes the following inputs:

- Major road flows.
- Minor road flows.
- Controlling flow as a function of geometry and traffic flows of higher rank movements.
- Traffic structure and composition.
- Critical gaps as a function of traffic movement geometry.
- Traffic signs and major road speed.
- Visibility distances.
- Approach width and grade.
- Pedestrian flows.

SINGAPORE

A procedure to estimate unsignalized intersection capacities has also been reported by Turner et al. (1988). Conceptually, it completely coincides with the British method. However, the regression analysis has been evaluated for conditions and driver behavior in Singapore.

The form and denotation of the equations correspond to equations 6.1 - 6.3. However, the terms containing the lane width available to the nonpriority streams have been dropped, because their contribution to capacity was found to be insignificant. Therefore, the following equations predict the capacity for one lane of the minor street.

$$\mu_{b-a} = D(645 - Y[0.365q_{a-c} + 0.115q_{a-b} + 0.230q_{c-a} + 0.587q_{c-b}]) \quad (12.1)$$

$$\mu_{b-c} = E(835 - Y[0.602q_{a-c} + 0.149q_{a-b}]) \quad (12.2)$$

$$\mu_{c-b} = F(833 - Y[0.381q_{a-c} + 0.350q_{a-b}]) \quad (12.3)$$

where

$Y = [1 - 0.035\ W]$

In each of these equations, the geometric parameters represented by D,E and F are stream-specific:

$D = (0.892 + 0.0009V_r)(0.912 + 0.0006V_l)$
$E = (0.895 + 0.0009V_r)$
$F = (0.895 + 0.0007V_{c-b})$

In all cases, demand volumes and capacities are in pcu/h and distances in meters. Capacities can only be positive or zero. If the right hand side of any equation is negative, then capacity is zero.

The equations also assume separate lanes for the right-turning and left-turning minor road traffic and that the straight through major road traffic volume q_{c-a} is unimpeded by the right-turning major road traffic, q_{c-b}.

μ = potential capacity (veh/h)
q = traffic volumes (veh/h) (denotation of movements: cf. Figure 9)
W = width of major street (m)
V = visibility (r: to the right; l: to the left)

SWEDEN

The Swedish method is based on the premise that there are two service rates. The first is for those minor stream vehicles which arrive when there is a queue and, in this case, the service time is equal to the inverse of the gap acceptance capacity. The relationship used is the same as eq. 2.4.1. The second group of minor stream drivers is that which arrives when there is not a queue on the minor stream approach. The service time is an example of Adams' delay as calculated by Troutbeck (1986). The average service time is then:

$$\bar{b} = B * b_d + (1 - B) * b_u \quad (14.1)$$

where

b_u is the undelayed service time
b_d is the delayed service time ($b_d = 1/Qe$)
B is the proportion of delayed vehicles

As the flows increase, the b_d term dominates and b approaches b_d.

Values for the critical gap and move-up times were evaluated at 18 locations and used in the analysis (Hansson, 1978).

The variance of the service times is then estimated and used to give the C term in the Pollaczek-Chintchine formula (eqn 2.5.6). This latter equation is then used to estimate delay.

The means of estimating the impedance of the minor streams uses a similar approach to the German technique and is based on the probability that there is no queue in the intermediate priority streams.

The approach allows for the service time to be adjusted for the effects of:

- Short lanes;

- Shared lanes;
- Pedestrians and bicycles;
- Wide medians; and
- Signal coordination.

The performance measures include:

- Capacity and degree of saturation;
- Delay and stops; and
- Queue lengths.

Additional material can be found in Hansson (1978), Hansson (1987), Anveden (1988), Hansson and Bergh (1988).

UNITED STATES OF AMERICA

In the United States, the HCM (1985), chapter 10, has introduced a method for the estimation of capacity and level of service at unsignalized intersections. It is based on the former German method and thus on gap acceptance theory.

Since the reader of this paper is expected to be quite familiar with this procedure, more details or comments are not provided. For an understanding of the procedure, however, the paper by Baass (1987) could be helpful.

NOTATIONS

D	=	average delay to nonpriority vehicles	(s)
D_q	=	average delay of vehicles in a queueing system excluding the service time	(s)
L	=	average queue length	(veh)
n	=	volume of a nonpriority stream (q_n /3600)	(veh/s)
p	=	volume of a priority traffic stream (q_p /3600)	(veh/s)
p_o	=	probability of no vehicle queueing in the nonpriority stream	(-)
q_e	=	potential capacity for minor street movements of rank 3 and 4. For rank-2-movements $q_e = q_{e,max}$	(veh/h)
$q_{e,max}$	=	potential capacity of the minor street entry which can only be achieved with a steady queue on the minor street approach	(veh/h)
q_m	=	capacity of a "shared lane"	(veh/h)
q_n	=	volume of a nonpriority traffic stream approaching the intersection	(veh/h)
q_p	=	volume of a priority traffic stream	(veh/h)
S_a^2	=	variance of the distribution of accepted gaps	(s^2)
T	=	duration of the peak period	(h)
t_a	=	accepted gap (gap in the priority traffic stream accepted by a nonpriority vehicle)	(s)
t_c	=	critical gap	(s)
t_f	=	follow-on time (=move-up time)	(s)
t_m	=	headway between bunched vehicles	(s)
W	=	average time which a vehicle spends in the first position of the queue	(s)
x	=	degree of saturation ($=q_n/q_{e,max}$)	(-)
γ	=	service rate ($q_{e,max}$ / 3600)	(veh/s)
α	=	proportion of free vehicles	(-)

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