2. REPRESENTING TRAFFIC FLOW AT A SIGNALIZED INTERSECTION

Learning objectives:

- Describe traffic flow characteristics on an intersection approach
- Apply a time-space diagram to describe flow parameters
- Describe the operation of a signalized intersection as a queuing process
- Represent the operation and performance of a signalized intersection using a flow profile diagram, a cumulative vehicle diagram, and a queue accumulation polygon

A model is often used in engineering to represent the behavior of a physical system. The model includes only those elements of the system that are relevant to the problem of interest. A model requires a set of input parameters that specify the state of the system. The model then produces a set of output parameters that help the engineer to evaluate the system performance based on the input state. In this section, a queuing model is used to represent traffic flow at a signalized intersection. Based on the state of the system (as represented by traffic volumes and signal timing parameters), the queuing model produces performance measures such as delay and queue length.

2.1 Vehicle Trajectories at a Signalized Intersection

The flow of vehicles approaching and traveling through a signalized intersection can be represented by a time-space diagram. A time-space diagram shows the position of each vehicle at any point in time, and the slopes of its trajectories show the speed of a vehicle at any point in time. The vehicular signal display intervals of green and red are also shown. The time that it takes to cycle through the display of these intervals is called the *cycle length*.

The time-space diagram in Figure 1 shows the flow of individual vehicles traveling through a signalized intersection (blue lines from lower left to upper right in the figure) during two complete signal cycles, illustrating several important concepts.

- During the first cycle, three vehicles arrive at the intersection during the green indication and travel through the intersection without stopping. The *arrival headway* h_a (the headway[1] between vehicles arriving at a signalized intersection) is constant, a flow pattern called *uniform flow*. We will assume the condition of uniform arrival flow in the queuing models that will be considered shortly.
- During the second cycle, three vehicles arrive at the intersection during the red indication. A fourth vehicle arrives during the green indication but must initially stop because the vehicle in front of it is stopped. As each of these four vehicles arrives, a queue (waiting line) forms at the stop line. The trajectory of the four vehicles is horizontal during red, illustrating that while time passes, the vehicle positions are stationary. The position of each vehicle

[1] *Headway* is defined as the time between the passage of successive vehicles at a given point.

in the queue, and the spacing between vehicles in the queue, is shown in the time space diagram.

- At the beginning of the green indication during cycle 2, the vehicles begin to enter the intersection. The headway between vehicles in the departing queue is called the *saturation headway* h_s.
- A final vehicle arrives during cycle 2 after the queue has cleared and travels through the intersection without stopping.

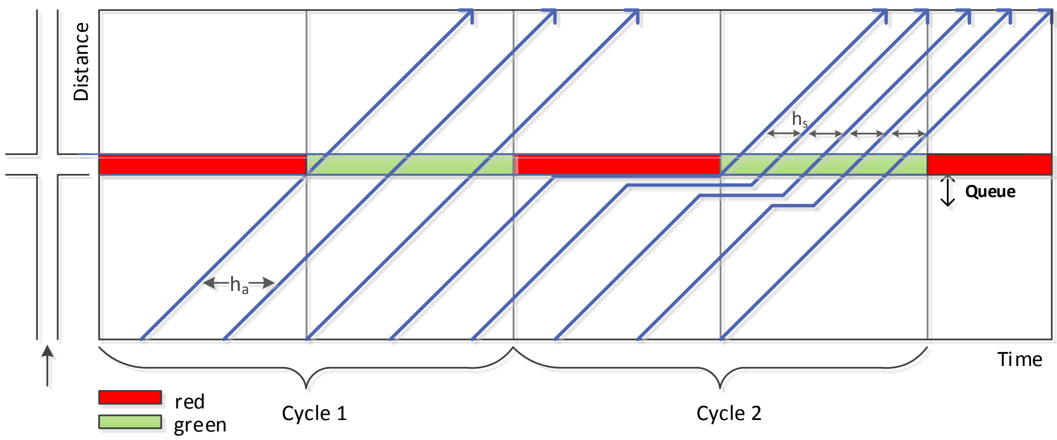

Figure 1. Vehicle trajectories (blue lines) represented in a time-space diagram

2.2 The Queuing Process at a Signalized Intersection

Traffic flow at a signalized intersection can be represented by a standard queuing model, known as the *D/D/1 model*. The D/D/1 model assumes a deterministic arrival pattern, a deterministic service pattern, and one service channel. A deterministic arrival or service pattern means that the pattern is known and does not vary over time. Stated another way, there is no randomness in the pattern. One service channel implies one lane of an intersection approach. The model also assumes that the demand is less than the capacity.

Figure 2 shows these elements as applied to one lane on an approach to a signalized intersection. The *server* is the first vehicle position at the stop bar. This is the point at which vehicles are served as they exit the queuing system and enter the intersection. The *queue* forms behind the vehicle in the server and extends to some maximum point, depending on the arrival and service rates of the system. While queuing theory assumes that this line of vehicles is a vertical stack, in reality the queue extends horizontally upstream from the stop bar to the point of the maximum queue. This point is considered to be the entry point to the queuing system.

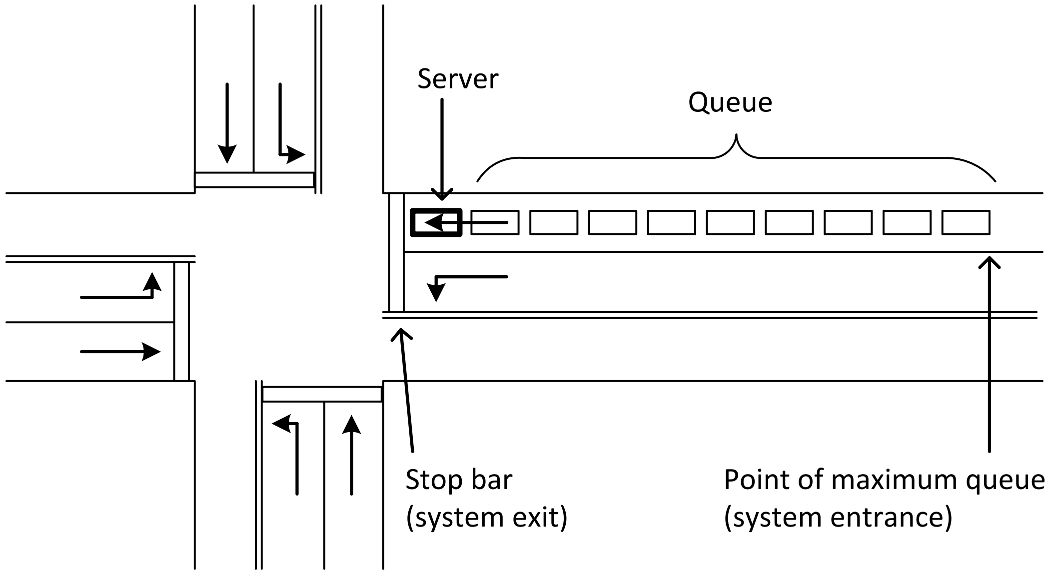

Figure 2. Elements of queuing system

Figure 3 shows another view of the queuing process, here overlaid vertically on a time-space diagram (at the left side of the figure). The arrival pattern is shown just upstream of the signalized intersection, showing the constant headway between each of the vehicles. As noted above, this pattern is called *uniform arrivals* and is consistent with the deterministic arrival pattern for the D/D/1 queuing model. The service or departure pattern is shown just downstream of the intersection for three time periods.

- During period 1, vehicles 1, 2, and 3 arrive during the red interval and form a queue. The service or departure rate is zero because the signal indication is red.
- When the signal display changes to green, vehicles 1, 2, and 3 depart from the intersection at the saturation flow rate. Here the headway between vehicles is equal to the saturation headway. A fourth vehicle arrives during green but joins the queue and can't be served until the queue clears. The service rate for this vehicle is also equal to the saturation flow rate. The time that it takes for the queue to clear (called the *queue service time* g_s) is the duration of the second period.
- During the third period, vehicles 5 and 6 arrive and depart at a constant rate, equal to the arrival rate. Since the queue has cleared, these vehicles experience no delay.

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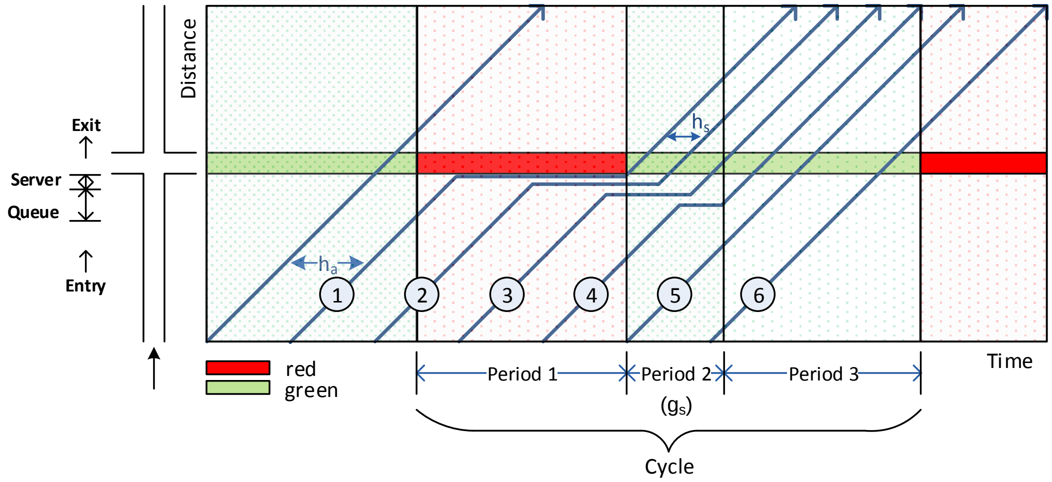

Figure 3. Vehicle trajectories arriving at and departing from a signalized intersection

There are three other ways to represent the queuing process including the flow profile diagram, the cumulative vehicle diagram, and the queue accumulation polygon. These diagrams also show important concepts in the traffic flow process such as capacity and delay, as well as other ways to represent intersection operation and performance, and are discussed below.

2.3 The Flow Profile Diagram

The *flow profile diagram* (Figure 4) represents both the arrival flow to and the departure flow from a signalized intersection over time. The flow profiles can be extracted from the time-space diagram shown in Figure 3 either by calculating the flow rate from the headway between vehicles, or by counting the number of vehicles over constant time segments. The arrival flow rate is constant (uniform) and is represented by the solid line in Figure 4. The service flow rate (represented by the dashed line) varies during the cycle, according to the three time periods noted in the discussion of the time-space diagram above. The service flow rate is equal to:

- Zero, during the red indication.
- The saturation flow rate *s*, while the queue is clearing, during an interval called the queue service time g_s.
- The arrival flow rate *v*, after the queue clears and until the end of the green interval.

Since the queue clears before the end of green, the volume is less than the capacity. This meets one of the assumptions of the D/D/1 queuing model established earlier.

This service pattern is repeated for each signal cycle.

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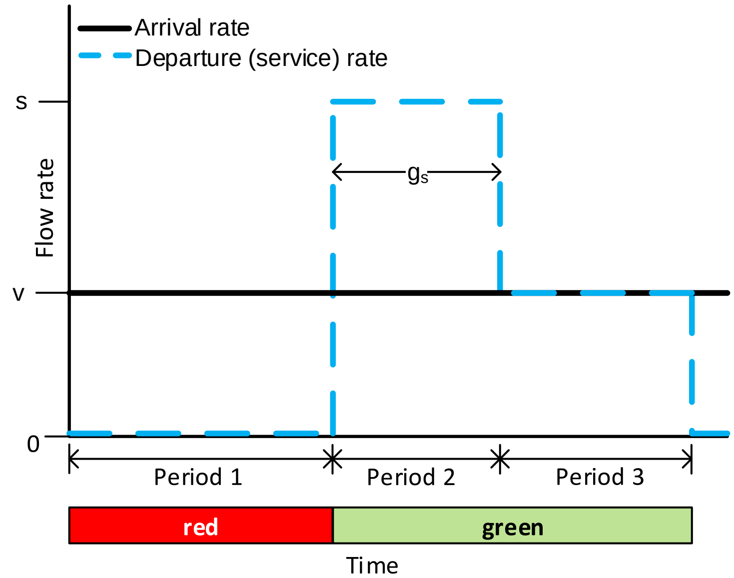

Figure 4. Flow profile diagram

Figure 5 shows that the server can be represented by a pipe with the arrival (input) flow profile diagram shown on the left and the service (output) flow profile shown on the right. The diameter of the pipe approximates the saturation flow rate.

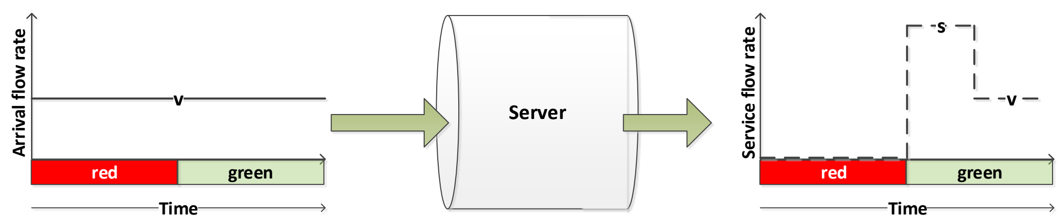

Figure 5. Queuing process showing input, server, and output

Example 1. Flow Profile Diagram

Prepare a flow profile diagram that represents the following conditions:

- Arrival flow rate = 600 veh/hr
- Saturation flow rate = 1900 veh/hr
- Queue service time = 13.8 sec
- Cycle length = 60 sec
- Green time = 30 sec
- Red time = 30 sec

The arrival flow profile is represented by a horizontal line whose value is a constant 600 veh/hr. The departure or service flow profile is represented by three line segments, one for each of the three periods described above:

- During red, the departure flow rate is zero.
- During the period that the queue is clearing, or the queue service time, the departure flow rate is equal to the saturation flow rate, or 1900 veh/hr.

- After the queue has cleared, the departure rate is equal to the arrival rate, or 600 veh/hr.

Figure 6 shows both the arrival and departure flow profiles for the conditions just described.

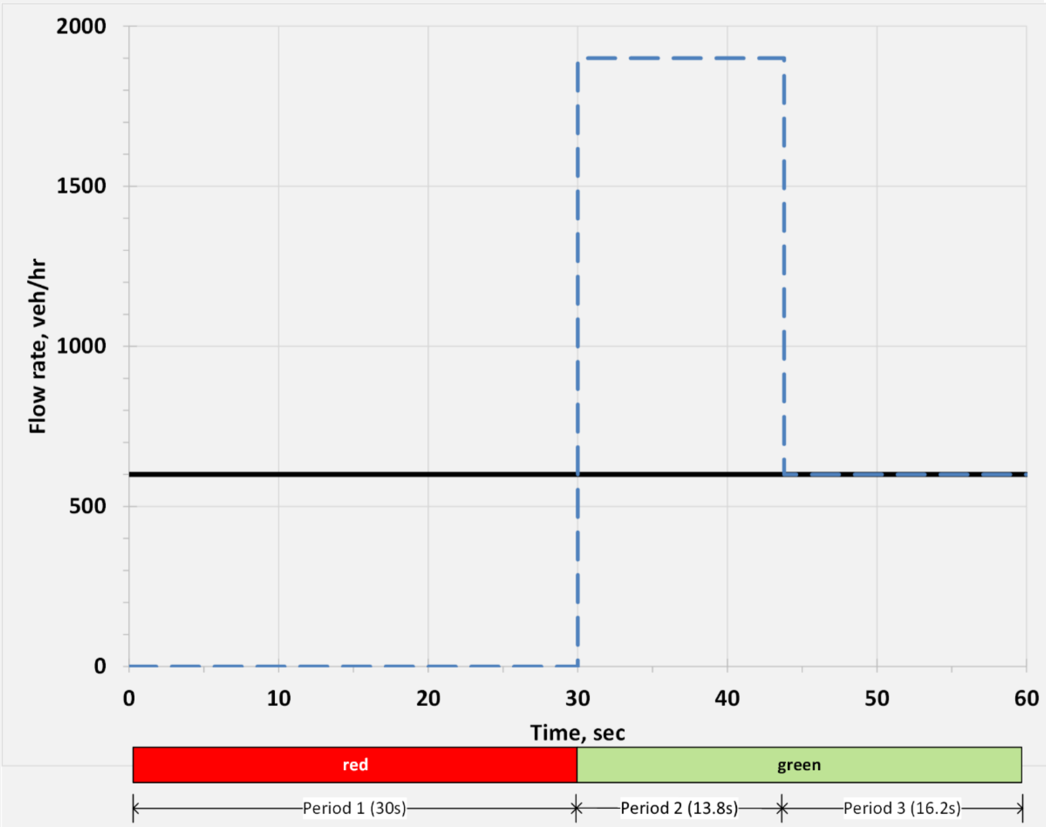

Figure 6. Example flow profile diagram

2.4 The Cumulative Vehicle Diagram

The *cumulative vehicle diagram* (Figure 7) is a running total of the number of vehicles that have arrived at and departed from the intersection over time. The cumulative vehicle diagram shows two lines, one representing the cumulative number of arrivals over time and the other the cumulative number of departures.

When we assume that the arrival pattern is uniform, with constant headways, the horizontal line from the flow profile diagram becomes a line of constant slope in the cumulative vehicle diagram. The slope of the line representing vehicle arrivals is equal to the arrival flow rate. As we move from left to right in the diagram (representing the passage of time), the y-axis value shows the running total of the number of vehicles that have arrived at the intersection at any point in time.

The service pattern is again divided into three periods. During the red indication, no vehicles can depart from the intersection, so the total number of departures is zero during this period. Once the green is displayed, vehicles begin to depart from the intersection at the saturation flow rate, with headways between vehicles equal to the saturation headway. The slope of the departure line during this time interval is equal to the saturation flow rate. At the point that the queue clears, the arrival line and the departure line become coincident. Their slopes are the same and equal to the vehicle arrival rate during the remainder of the green interval.

The cumulative vehicle diagram also shows three measures of intersection performance, indicating how well the intersection is operating.

- The first measure is the length of the queue at any point in time. The *length of the queue* is the difference between the number of vehicles that have arrived at and departed from the intersection at any point in time. Graphically, the queue length (or number of vehicles currently in the system, Q) is the vertical distance between the arrival line and the departure line, as illustrated in Figure 7.

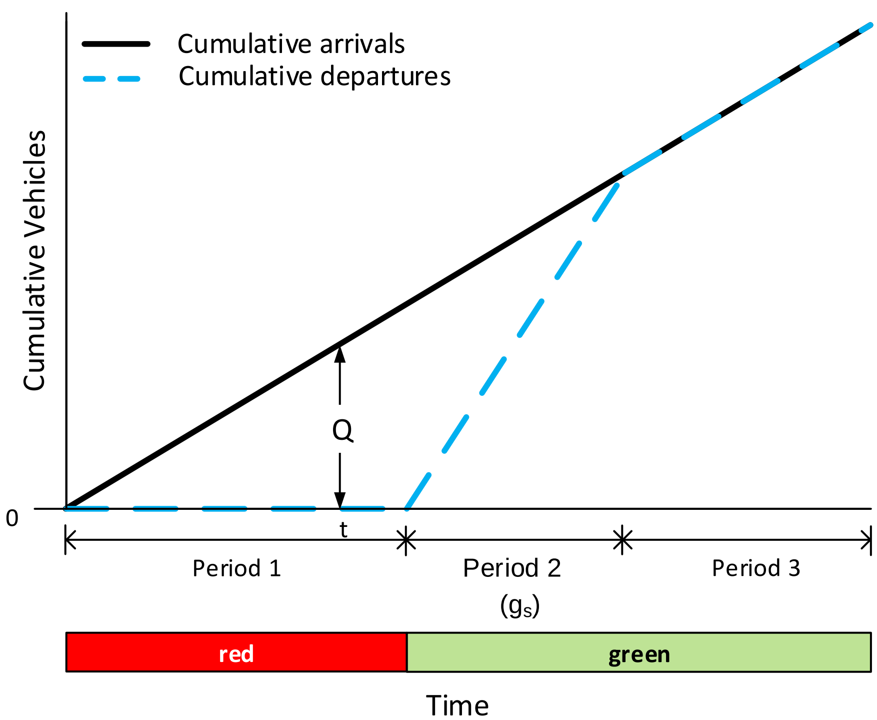

Figure 7. Cumulative vehicle diagram showing queue length at time t

- The second measure is the time that each vehicle spends in the system, or the *delay* that it experiences. Consider the horizontal line connecting the arrival line and the departure line for the vehicle (noted as vehicle i) shown in Figure 8. Point 1 on the arrival line is the time that the vehicle enters the system; point 2 on the departure line is the time that the vehicle exits the system. The time interval between these two points is the delay experienced by the vehicle.

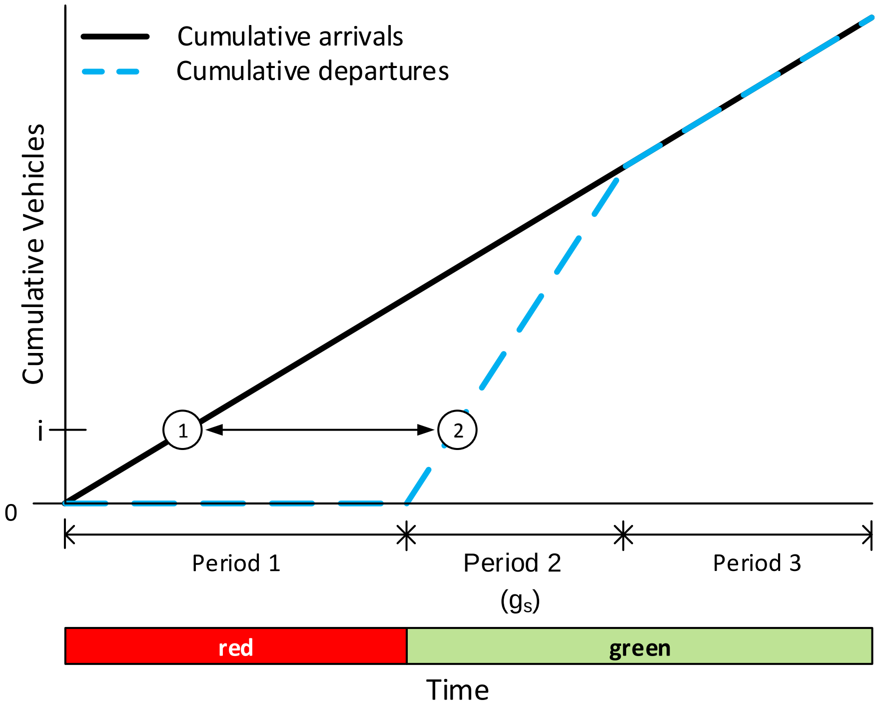

Figure 8. Cumulative vehicle diagram showing delay for vehicle i

- The third measure is the *total delay* experienced by all vehicles that arrive at and travel through the intersection. If we add all of the horizontal lines described in the bullet above for all vehicles, we get the total delay experienced by all vehicles. The total delay is the area of the triangle formed by the arrival and departure lines and shown as the shaded area in Figure 9.

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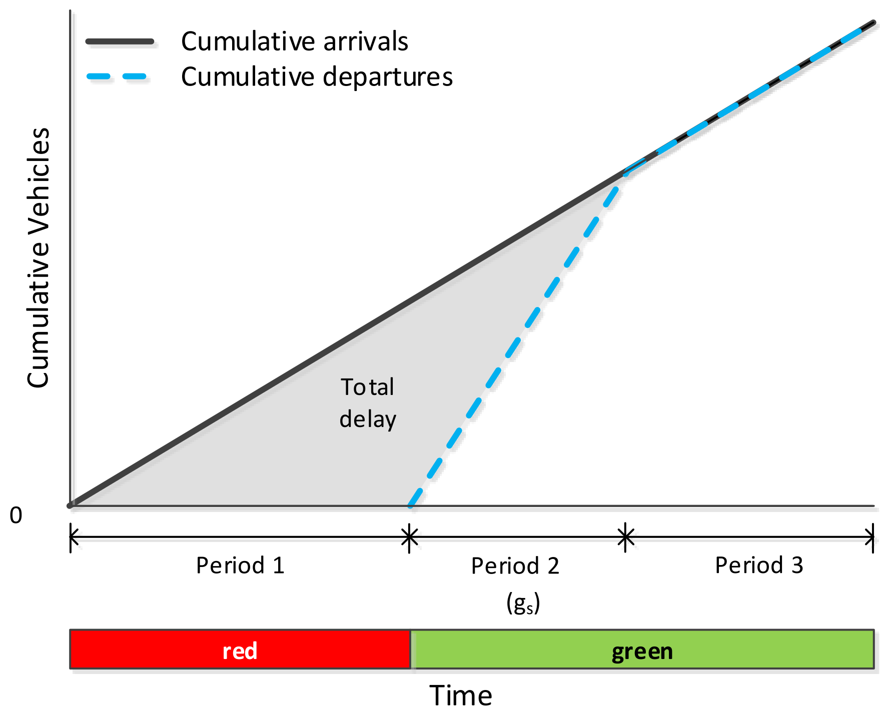

Figure 9. Cumulative vehicle diagram showing total delay

Example 2. Cumulative Vehicle Diagram

Field data collected on one approach of a signalized intersection showed that:

- Vehicles arrive every 6 sec at a uniform rate.
- The cycle length is 60 sec, with red and green time intervals of 30 sec each.
- Vehicles depart every 2 sec after the beginning of green.
- The queue service time is 14 sec.

Prepare a cumulative vehicle diagram that represents these conditions.

The cumulative vehicle diagram for these conditions is shown in Figure 10. Since a vehicle arrives at the intersection every six seconds, a cumulative total of five vehicles arrive from the beginning of red until the end of red. We've assumed that the vehicle that arrives at t = 0 (end of green) travels through the intersection without stopping. Also, since we are dealing with the discrete events of vehicle arrivals in the field, the lines are "stair step" with each increase representing a vehicle arrival or departure. By contrast, the theoretical depictions presented earlier are based on a continuous and not discrete process.

The chart shows that at t = 44 sec, the cumulative number of vehicles that have entered the system (seven) equals the number that have exited. It is at this point (14 sec after the start of the green interval) that the arrival and departure lines become coincident and the queue clears.

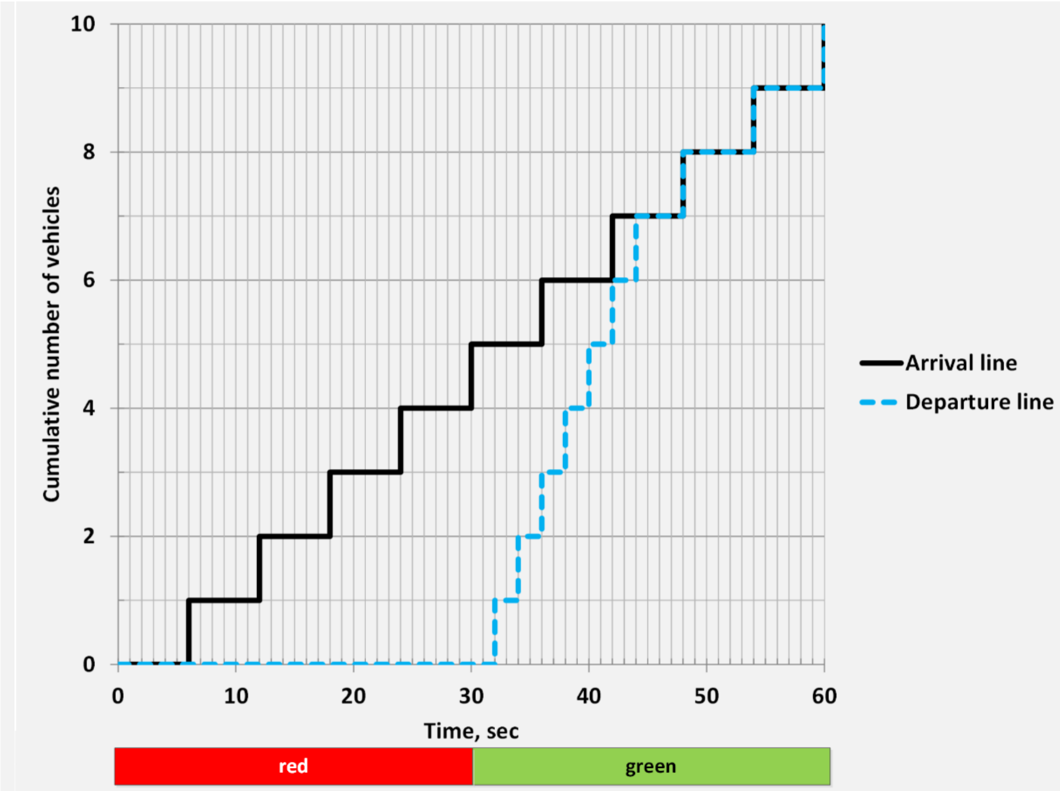

Figure 10. Example cumulative vehicle diagram

What is the delay for the vehicles that arrive at the intersection? Table 1 shows the times that each vehicle arrives at and departs from the intersection, as read from the cumulative vehicle diagram (Figure 10) for the first seven vehicles. The difference between the arrival and departure times is the time in the system or the delay experienced by each vehicle.

Table 1. Vehicle arrivals and departures (and time in system)

Vehicle number	Arrival time (sec)	Departure time (sec)	Delay (time in system), (sec/veh)
1	6	32	26
2	12	34	22
3	18	36	18
4	24	38	14
5	30	40	10
6	36	42	6
7	42	42	2
		Total	98 sec

Vehicle #1 has the longest delay (26 sec) since it arrives near the beginning of the red interval. The seventh vehicle arrives just as the queue is clearing and has a delay of about 2 sec. Vehicles 8, 9, and 10 arrive and leave without delay, as the queue has cleared by the time that vehicle 8 arrives. The total delay for all vehicles is 98 sec.

2.5 The Queue Accumulation Polygon

The *queue accumulation polygon* represents the length of the queue at any point in time and is derived from the cumulative vehicle diagram. Figure 11 shows the queue accumulation polygon, again for the case of uniform arrivals: the queue grows during red and reaches its maximum length at the end of the red interval (or the beginning of the green interval). It decreases once the green interval begins and reaches zero when the queue clears. It remains at zero until the end of the green interval.

The area of the queue accumulation polygon is equal to the area between the arrival and departure lines of the cumulative vehicle diagram: both areas represent the total delay experienced by all vehicles that arrive and leave during the cycle.

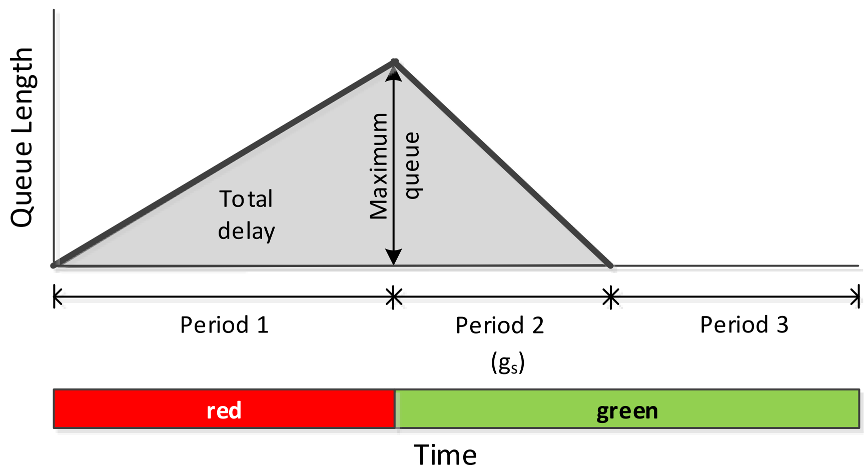

Figure 11. Queue accumulation polygon

Example 3. Queue Accumulation Polygon

Consider the conditions given for the cumulative vehicle diagram in Example 2. Prepare a queue accumulation polygon that represents these conditions.

The queue accumulation polygon represents the length of the queue over time and is shown in Figure 12 for the given conditions. The queue accumulation polygon shows the queue growing during red and reaching a maximum length of 5 vehicles at the end of red (t = 30 sec). The queue begins to clear when the display changes from red to green and clears 14 sec later (t = 44 sec). The queue remains at zero after t = 44 sec, as vehicles arrive and depart without delay. Figure 12 represents the discrete form of the queue accumulation polygon where the change for each vehicle is shown, in contrast to the continuous form shown in Figure 11.

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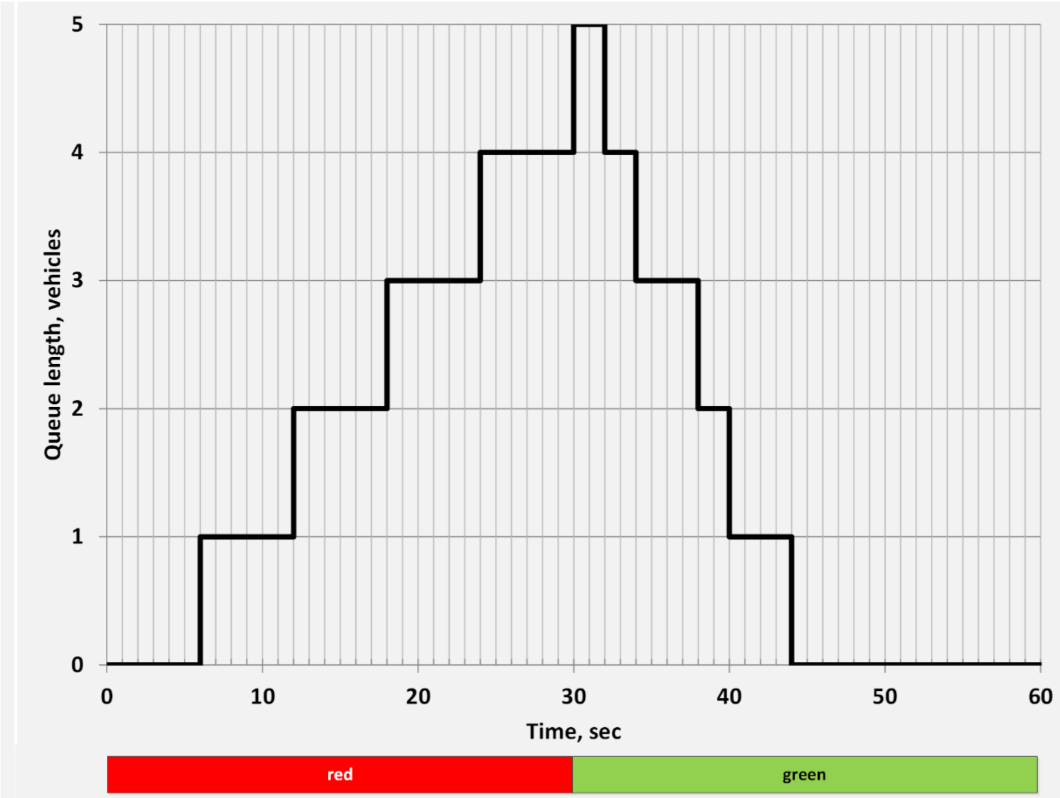

Figure 12. Example queue accumulation polygon

2.6 Summary of Section 2

What You Should Know and Be Able To Do:

- Describe traffic flow characteristics on an intersection approach
- Apply a time-space diagram to describe flow parameters
- Describe the operation of a signalized intersection as a queuing process
- Represent the operation and performance of a signalized intersection using a flow profile diagram, a cumulative vehicle diagram, and a queue accumulation polygon

Concepts You Should Understand:

- Concept 2.1: Queuing process at signalized intersection

We have represented traffic flow on one lane at a signalized intersection as a queuing process. The arrival pattern consists of uniform flow with a constant rate. The service pattern is represented by three values: zero during red, the saturation flow rate during the queue clearance process, and the arrival flow rate after the queue has cleared. The process can be represented by a flow profile diagram.

- Concept 2.2: Queuing process representation

The queuing process can also be graphically represented as a cumulative vehicle diagram and a queue accumulation polygon. Each diagram shows the evolution of the queue length during the cycle (the growth during red and the clearing during green) and the total delay experienced by all users of the system (the areas of the triangles).