CHAPTER 5. TWO-WAY STOP-CONTROLLED INTERSECTIONS

1. Introduction/Overview
[modify]
A two-way stop-controlled (TWSC) intersection provides priority to vehicles on the major street while requiring vehicles on the minor street to stop before entering the intersection. Minor street vehicles look for gaps in the major traffic stream that are large enough for them to safely enter the intersection and complete their maneuver. In this chapter we will explore the models on which the HCM operational analysis method for TWSC intersections is based.

The operation of a TWSC intersection is based on a process called gap acceptance. The gap acceptance process is a common one in traffic operations. Consider for example:
- Vehicles merging in a freeway on-ramp to the mainline.
- Vehicles passing on a rural two-lane highway.
- Vehicles making a permitted left turn at a signalized intersection.
- Minor street vehicles at a TWSC intersection.

We will study the latter two items as we construct our models of intersection operations. We will start with a brief review of probability theory (the Poisson distribution and the negative exponential distribution), then apply this theory to the gap acceptance process, and finally to consider other aspects of traffic stream interaction such as impedance to construct a complete model of TWSC intersection operations.

Why do we need to study TWSC intersections? Issues for discussion:
- How do we know what the proper intersection control should be?
- When is it justified or warranted to change from stop sign control to signal control?
- Is driveway or exit capacity sufficient (or will sign control work for exit at new residential development)?
- What are the optimal “performance” regions: region of no control (low volume), TWSC (low minor street), AWSC (evenly balanced), roundabout, signal control (higher volumes).

The chapter begins with a discussion of two simplified scenarios of an AWSC intersection so that you can more clearly see that theory on which the HCM operational analysis method is based. The first scenario is the intersection of two one-way streets, with one lane on each approach. The second scenario is based on a four-leg intersection, again with one lane on each approach. In both of these scenarios, the concept of the degree of conflict faced by vehicles on each approach is described. To understand the workings of the HCM method under these scenarios, you will build and validate a computational engine, a spreadsheet tool that will allow you to quickly see the results of various traffic flow conditions on intersection operation. This will allow you to see how an AWSC intersection is predicted to perform and under what conditions it will reach capacity. [Delay?] Finally, we will introduce the components of the full HCM method to account for turning movements and multilane approaches.

2. Availability of Gaps

Poisson and negative exponential distributions

Two common statistical distributions are used to represent random phenomenon:

- The Poisson distribution is a discrete distribution, often used to describe how many random events will occur during a specific time interval.
- The negative exponential distribution is a continuous distribution that is used to represent the time between the occurrences of these random events.

The number of vehicles arriving at a point and the headway between these vehicles are the traffic phenomenon that may be represented as random, if the arrival of each vehicle is an independent event (no vehicle interaction).

Suppose we have a single lane on which traffic is flowing in one direction. Further assume that the operation of each vehicle is independent of any of the other vehicles on the roadways. If the mean flow rate is λ (veh/sec), the probability of observing x vehicles during a specified time period t is:

Equation 1

$$P[x] = \frac{(\lambda t)^x}{x!} e^{-\lambda t}$$

What is the probability that we observe no vehicles during the interval t?

Equation 2

$$P[x = 0] = \frac{(\lambda t)^0}{0!} e^{-\lambda t} = e^{-\lambda t}$$

Another way of looking at this situation is, what is the probability that we will see a gap or headway in the traffic flow of at least t seconds?

$$P[t \geq t_c] = e^{-\lambda t}$$

This is the negative exponential distribution.

3. Gap Acceptance Model

Scenario #1

Tc and tf

Availability and usefulness of gaps

Gap capacity

Harders model

Let's consider the intersection of two one-way streets, with the traffic of one street having priority over the traffic on the other street.

[sketch showing major street and priority stream, and minor street and non-priority stream]

Three basic elements in the gap acceptance process:
- Extent to which minor stream drivers find major stream gaps of a particular size to be useful.
- Proportion of gaps of a particular size in the major stream that are offered to minor stream driver.
- Interaction between streams in which we consider a hierarchy among competing movements.

We'll first consider the first two of these bullet points.

Extent to which minor stream drivers find gaps to be useful:
- Behavior of minor stream vehicles is represented by two variables: critical gap and follow up time (t_c and t_f).
- Critical gap = minimum time gap in the priority traffic stream required by minor stream driver to enter the major traffic stream.
- Follow up time = minimum time headway between two consecutive minor stream vehicles entering the major stream (and using the same major stream gap) during conditions of continuous queuing on the minor street.
- We assume that:
    - Drivers are homogeneous: all drivers behave the same way at any location.
    - Drivers are consistent: drivers behave the same way every time.
    - Difficulty in measuring t_c and t_f: we can only measure tf directly.

Numerical example: Can we compute the capacity of a minor stream, knowing tc and tf?
For a minor TH movement, tc = 6.5 sec and tf = 4.0 sec.

Gap size (sec)	**Number of vehicles absorbed**
$t_g < 6.5$	0
$6.5 \leq t_g \leq 10.5$	1
$10.5 \leq t_g \leq 14.5$	2
$14.5 \leq t_g \leq 18.5$	3
$18.5 \leq t_g \leq 22.5$	4

Distribution of gap sizes in priority streams:
- What is the distribution of gaps that are actually available to the minor stream driver?
- For many conditions we can start by assuming a random arrival pattern. This means that the interarrrival times follow an exponential distribution:

Probability of x events during t.

$$P[x] = \frac{(\lambda t)^x}{x!} e^{-\lambda t}$$

Probability of no events during t, or a headway between events of at least t.

$$P[x = 0] = \frac{(\lambda t)^0}{0!} e^{-\lambda t} = e^{-\lambda t}$$

$$P[t \geq t_c] = e^{-\lambda t}$$

$$F[t] = 1 - e^{-\lambda t}$$

Let's combine both elements (driver needs and available gaps) to compute the capacity of a stream. We can use the following table for this computation:

Gap size, t_g	**Number of vehicles absorbed**	**Probability of gap occurrence**
$t_g < t_c$	0	$1 - e^{-\lambda t_c}$
$t_c \leq t_g \leq t_c + t_f$	1	$e^{-\lambda t_c} - e^{-\lambda(t_c - t_f)}$
$t_c + t_f \leq t_g \leq t_c + 2t_f$	2	$e^{-\lambda(t_c - t_f)} - e^{-\lambda(t_c - 2t_f)}$
…		
$t_c + (n-1)t_f \leq t_g \leq t_c + nt_f$	n	$e^{-\lambda(t_c - (n-1)t_f)} - e^{-\lambda(t_c - nt_f)}$

For general gap size (last row of table above):
The number of vehicles that can be absorbed during one hour:
Σ(total number of available gaps)(probability of gap of given size occurring)(number of vehicles than can use that gap)
Or,

$$= \sum_{0}^{n} V_c(n)[\, e^{-\lambda(t_c - (n-1)t_f)} - e^{-\lambda(t_c - nt_f)}]$$

As n →∞

$$c = \frac{v_c e^{-v_c t_c/3600}}{1 - e^{-v_c t_f/3600}}$$

How does capacity vary with major street flow?

When $V_c = 0$:

$$c \rightarrow \frac{3600}{t_f} = \frac{3600}{4.0} = 900\ veh/hr$$

V_c = 200 veh/hr (or mean headway = 18 sec)

$$c = \frac{v_c e^{-v_c t_c/3600}}{1 - e^{-v_c t_f/3600}} = \frac{200e^{-200(6.5)/3600}}{1 - e^{-200(4.0)/3600}} = \frac{139.3803}{.1993} = 699\ veh/hr$$

V_c = 900 veh/hr (or mean headway = 4 sec)

$$c = \frac{v_c e^{-v_c t_c/3600}}{1 - e^{-v_c t_f/3600}} = \frac{900e^{-900(6.5)/3600}}{1 - e^{-900(4.0)/3600}} = \frac{177.2295}{.6321} = 280\ veh/hr$$

[add chart showing capacity vs conflicting flow rate]

Other notes:
Now consider what happens at a TWSC intersection. Simple condition of two intersecting one-way streets. Drivers from the minor street who want to cross or merge with the major street traffic stream require some minimum time headway t_c.

[Charts showing intersection type and t_f from notes]

The gap acceptance process assumes the following conditions:

- Each driver has a constant critical gap tc that is the same for all drivers making similar maneuvers.
- For long gaps in the major street traffic stream, more than one minor stream vehicle can enter the stream. The headways between these vehicles is called the follow up time, tf.

We can use this information to construct a model for the capacity of the minor street.

We know that the probability of any gap in the major traffic stream being at least tc seconds is $e^{-\lambda t}$. If there are v gaps during one hour (assume the flow rate is v veh/hr), the capacity is:

$$c = ve^{-\lambda t_c} = \ ve^{-vt_c/3600}$$

But we know that some gaps are large enough for more than one vehicle. This is the probability that we will see a gap that is large enough for one vehicle to enter the traffic stream:

$$P[\mathrm{t}_c \ \leq \ \mathrm{h} \ \ \leq \ \mathrm{t}_c \ + \mathrm{t}_f \ ] = e^{-\lambda t_c} - e^{-\lambda(t_c - t_f)}$$

The probability that we will see a gap that is large enough to accommodate or absorb exactly n vehicles is:

$$P\left[\mathrm{t}_c \ + \ (\mathrm{n}-1)\mathrm{t}_f \ \ \leq \ \mathrm{h} \ \ \leq \ \mathrm{t}_c \ + \mathrm{n}t_f \right] = n\left(e^{-\lambda(t_c-(n-1)t_f)} - e^{-\lambda\left(t_c - nt_f\right)}\right)$$

This process can be summarized in the following table:

Gap size, t_g	**Number of vehicles absorbed (capacity)**	**Probability of gap occurrence**
$t_g < t_c$	0	$1 - e^{-\lambda t_c}$
$t_c \le t_g \le t_c + t_f$	1	$e^{-\lambda t_c} - e^{-\lambda(t_c - t_f)}$
$t_c + t_f \le t_g \le t_c + 2t_f$	2	$e^{-\lambda(t_c - t_f)} - e^{-\lambda(t_c - 2t_f)}$
…		
$t_c + (n-1)t_f \le t_g \le t_c + nt_f$	n	$e^{-\lambda(t_c-(n-1)t_f)} - e^{-\lambda(t_c - nt_f)}$

The capacity is the sum of the product of columns 2 and 3:

$$c = \frac{v_c e^{-v_c t_c/3600}}{1 - e^{-v_c t_f/3600}}$$

This is the basic equation for the capacity of a minor stream at a TWSC intersection that is given in chapter 19 of the HCM.