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# The Potential Capacity of Unsignalized Intersections

by Karsten G. Baass

The 1985 *Highway Capacity Manual* (HCM) bases the capacity analysis of unsignalized intersections on its Figure 10.3, which gives the potential capacity of the nonpriority traffic stream in relation to conflicting traffic movements.[1] The diagram is derived from the German method of capacity determination at rural intersections.[2] This article will describe the theoretical background leading to the establishment of this method so that its uses and limitations may be better understood.

The number of vehicles that can cross an intersection depends essentially on two variables: the number of acceptable gaps in the priority stream and the gap acceptance distribution of drivers in the secondary traffic stream. The negative exponential distribution allows us to estimate the probability of a gap of length $t$ in a traffic stream of volume $V$, if the hypothesis of Poisson arrivals can be made. This probability is determined using the following:

$$P(h \geq t) = e^{-Vt/3600}.$$

A driver needs a minimum gap of length $T_c$ to cross the street. Based on this assumption, Grabe calculates the probable number of vehicles that can cross a traffic stream consisting of two directions $V_1$ and $V_2$ ($V_1 + V_2 = V_c$).[3] The probability of finding a gap of length $T_c$ in the two conflicting traffic streams is given by the basic law of probability:

$$P(h \geq T_c) = P_1(h \geq T_c) \cdot P_2(h \geq T_c) = e^{\frac{-(V_1 + V_2)T_c}{3600}} = e^{-\alpha}.$$

Grabe, and subsequently Major and Buckley, supposes that each driver needs the same critical gap, $T_c$, to cross and this is independent of his position in the queue.[3,4] There would be no vehicle crossing for all gaps between 0 and $T_c$ of length, one vehicle would cross providing the gaps are between $T_c$ and $2T_c$, while two vehicles would be able to cross when there are gaps between $2T_c$ and $3T_c$ and so on. So there would be $n$ vehicles that could cross in gaps of a length between $nT_c$ and $(n+1)T_c$.

The probability of the presence of a gap between $nT_c$ and $(n+1)\,T_c$ is

$$P = e^{-n\alpha} - e^{(n+1)\alpha}.$$

The number of vehicles that can cross the conflicting traffic stream is determined by

$$N = n\,V_c\,(e^{-n\alpha} - e^{-(n+1)\alpha})$$

and given in Table 1.

The total number of vehicles able to cross the priority stream is obtained by finding the sum of column 3 in Table 1. This gives

$$S = V_c\,(e^{-\alpha} + e^{-2\alpha} + e^{-3\alpha} + \ldots + e^{-n\alpha}).$$

The sum of this geometric series is

$$C_p\,(n \rightarrow \infty) = V_c\,e^{-\alpha}\,\frac{1 - e^{-n\alpha}}{1 - e^{\alpha}} = \frac{V_c}{e^{\alpha} - 1}.$$

Harders develops this approach further.[5] Instead of using a constant critical gap for all drivers independent of their position in the queue, he supposes that the second and any cars further back in the queue need a lesser critical gap to cross, which he calls follow-up gap—$T_s$. These gaps were observed by Harders and reported for different types of traffic control methods employed at the intersection. For this case Table 1 is converted into Table 2 with

$$\beta = \frac{V_c\,T_s}{3600}.$$

Finding the sum of column 3 in Table 2 gives

$$S = V_c\,(e^{-\alpha} + e^{-\alpha}e^{-\beta} + e^{-\alpha}e^{-2\beta} + \ldots + e^{-\alpha}e^{-(n-1)\beta}).$$

The sum of this geometric series is

**Table 1. Probable Number of Vehicles Crossing in Gaps of Varying Lengths Assuming a Constant Critical Gap $T_c$**

| *Gap Between* | *Number of Vehicles Able to Cross in Gap* | *Probable Number of Vehicles in Gap* |
|---|---|---|
| 0 and $T_c$ | 0 | 0 |
| $T_c$ and $2T_c$ | 1 | $1\,(e^{-\alpha} - e^{-2\alpha})\,V_c$ |
| $2\,T_c$ and $3T_c$ | 2 | $2(e^{-2\alpha} - e^{-3\alpha})V_c$ |
| • | • | • |
| • | • | • |
| • | • | • |
| $(n-1)\,T_c$ and $nT_c$ | $(n-1)$ | $(n-1)\,(e^{-(n-1)\alpha} - e^{-n\alpha})V_c$ |
| $nT_c$ and $(n+1)\,T_c$ | $n$ | $n(e^{-n\alpha} - e^{(n+1)\alpha})V_c$ |

$$C_p = e^{-\alpha}\left[\frac{1 - e^{-n\beta}}{1 - e^{-\beta}}\right] V_c .$$

By letting $n$ approach infinity and multiplying the numerator and denominator by $e^{-\beta}$, one obtains, for the potential capacity

$$C_p = \frac{e^{-(\alpha-\beta)}}{e^{\beta} - 1} V_c .$$

This formula is more representative of reality than the one developed by Grabe.

Up until this point it was supposed that drivers adopt the same median critical and follow-up gaps. There is, however, a significant variation about these median values, and Harders considers that this should be taken into account when determining the potential capacity.[5] Figure 1 shows a typical example of a frequency distribution of the critical and the follow-up gaps (measured in Hannover, Germany).

The error one commits when using only the median values in the estimation of potential capacity instead of the probability distribution increases as main street volume increases. If one puts $C_p$ as the possible capacity, estimated using the median values only, and $K_p$ as the capacity estimated by considering the distribution of $T_c$ and $T_s$, one can define a parameter $f = K_p/C_p$. Harders evaluated this parameter theoretically and found that its influence is not negligible, but nevertheless would not justify

**Table 2. Probable Number of Vehicles Crossing Assuming a Critical Gap and a Shorter Follow-Up Gap.**

| *Gap Between* | *Number of Vehicles Able to Cross in Gap* | *Probable Number of Vehicles in Gap* |
|---|---|---|
| 0 and $T_c$ | 0 | 0 |
| $T_c$ and $T_c + T_s$ | 1 | $1\,(e^{-\alpha} - e^{-\alpha}e^{-\beta})\,V_c$ |
| $T_c + T_s$ and $T_c + 2T_s$ | 2 | $2(e^{-\alpha}e^{-\beta} - e^{-\alpha}e^{-2\beta})V_c$ |
| • | • | • |
| • | • | • |
| • | • | • |
| $T_c + (n-1)\,T_s$ and $T_c + nT_s$ | $n$ | $n\,(e^{-\alpha}e^{-(n-1)\beta} - e^{-\alpha}e^{-n\beta})V_c$ |

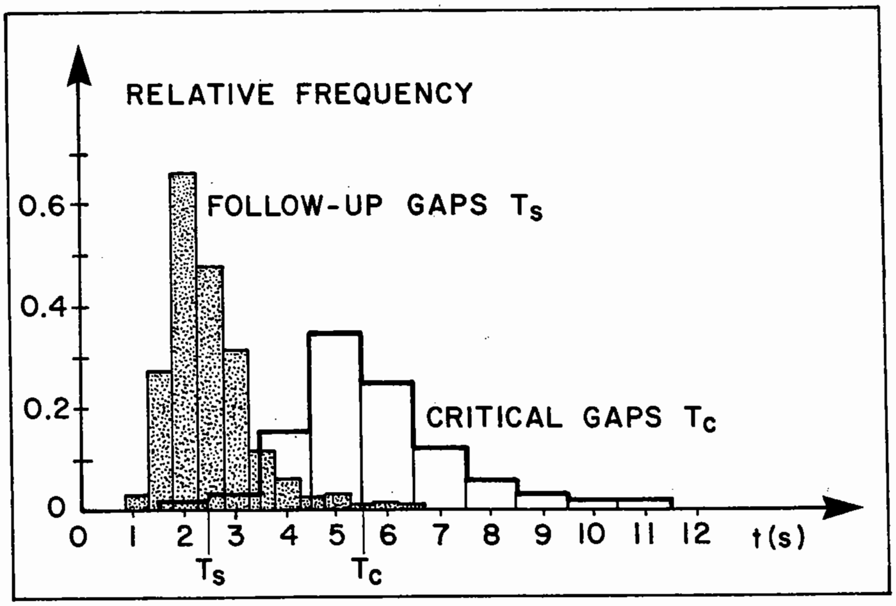


**Figure 1. Typical frequency distributions for critical and follow-up gaps. Source: Cited reference 5.**

a further complication of the theoretical formulae.[5] He observed the actual capacities $K_p$ at several intersections at different main street volumes and compared these with the values calculated using the preceding formula for $C_p$, which gave the values for $f$ in Figure 2.

Based on these observations he derived a simple relationship between $V_c$ and $f$, and the reduction of potential capacity can be given approximately by

$$f = 1 - 10^{-7} V_c^2 .$$

Thus, the formula that relates the main street volume to the potential capacity of the secondary street becomes

$$C_p = (1 - 10^{-7} V_c^2) \left[\frac{e^{-(\alpha-\beta)}}{e^{\beta} - 1}\right] V_c .$$

If the volume $V_c$ is zero then

$$\lim (V_c \rightarrow 0)\ C_p = \frac{3600}{T_s}\left[\frac{\beta}{e^{\beta} - 1}\right].$$

The expression between parentheses can be developed into a series that approaches 1 as β tends to 0.

$$\frac{\beta}{e^{\beta} - 1} = 1 - \frac{\beta}{2} + \frac{B_1 \beta^2}{2!} + - \ldots .$$

and

$$C_p \text{ (for } V_c = 0) = \frac{3600}{T_s} .$$

The points superimposed on Figure 10.3 of the 1985 HCM were calculated with the equation, choosing $T_s$ in such a way as to fit the curves as closely as possible (Figure 3).

It can be concluded that the formula adequately represents the curves except at volumes near 0 where lower potential capacities were supposed. These differences at low volumes are less important, because main street volumes are generally greater than side street volumes. The equation could, consequently, be used instead of Figure 10.3 of the HCM.

If one relates the follow-up gaps as obtained on Figure 3 to the critical gaps, one obtains a linear relationship between these two variables. This relation is given in Figure 4.

The relation between $T_s$ and $T_c$ influences the potential capacity. Figure 5

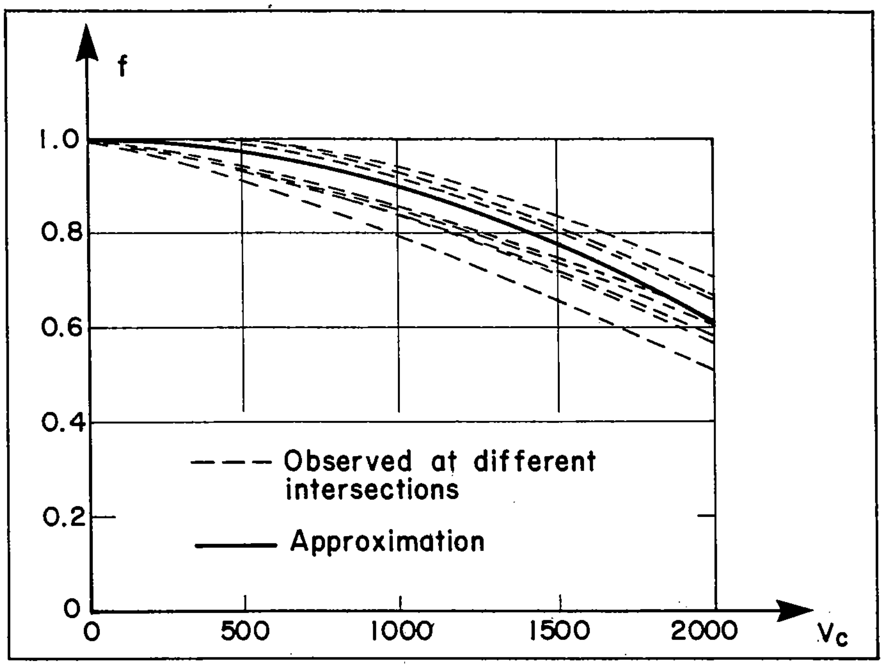


**Figure 2. Reduction of potential capacity with respect to increase in volume (observed in Hannover, Germany). SOURCE: Cited reference 5.**

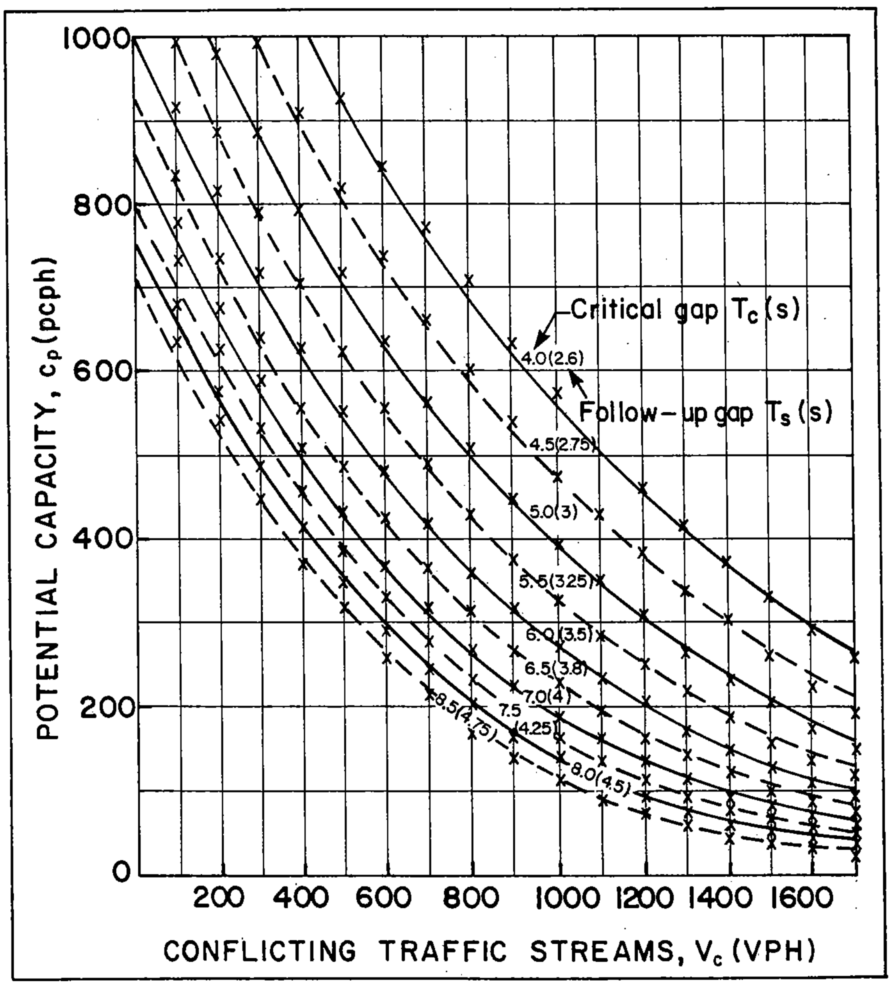


**Figure 3. Points calculated by the equation superposed on Figure 10.3 of the 1985 HCM.**

shows, as an example, the potential capacity (calculated with the formula) for a critical gap of 5 seconds together with follow-up gaps of 2.5, 3 and 3.5 seconds.

The variation of potential capacity obtained for a variation of the follow-up gap, of plus or minus 0.5 seconds, is quite important, at least in the low-volume range. It seems, therefore, necessary to verify the relationship between critical gaps and follow-up gaps in a North American context, because driver behavior might be different.

Some modifications to the existing German guidelines have been proposed in recent years. For example, Brilon considers that even if there are certain shortcomings in the methodology, it is useful "for easy numerical evaluation of the rather complex interrelations between traffic streams at an unsignalized intersection. This is the reason why also in the future the framework of this procedure should be accepted. However, there are still some drawbacks, which demand for some improvements."[6]

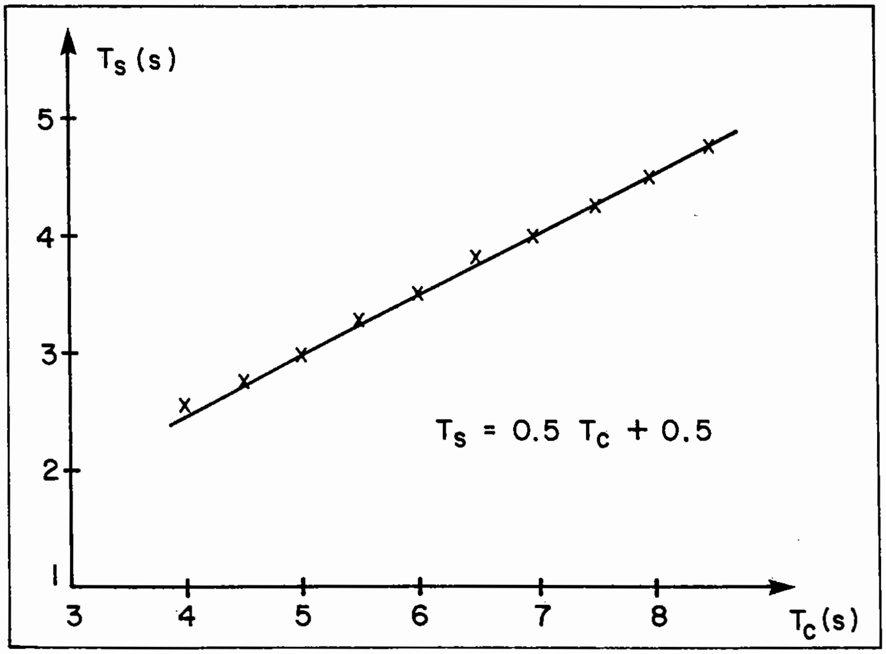


**Figure 4. Relationship between critical gap and follow-up gap.**

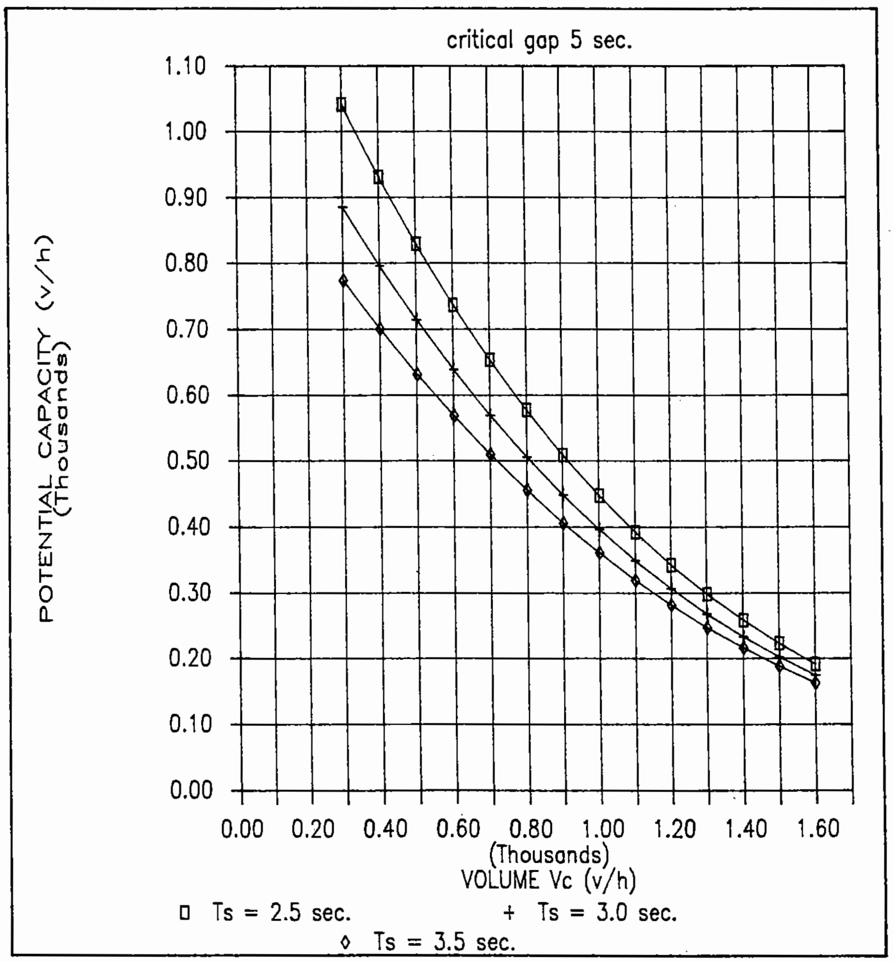


**Figure 5. Potential capacity calculated for a critical gap of 5 seconds and follow-up gap of 2.5, 3, and 3.5 seconds.**

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