1. INTRODUCTION

What is simulation? Consider two definitions:

- "To assume the appearance of without the reality"
- "Imitation of a real situation by some form of model"

Why is simulation an important tool? Often we need to study some process to see how it will react to changing conditions. We can sometimes build a test model and see how it will react to different input conditions. But this can often be expensive, impractical, or just not possible to do.

Consider the design of an aircraft. Aerospace engineers have for years built scale models of alternative designs and tested how they "fly" in a wind tunnel. This is certainly less expensive and safer than building each new alternative aircraft design, flying it, and seeing how effective the design is. But there is an even better way now, with the advent of supercomputers. Engineers can design and "build" a model of aircraft, and then using the equations of fluid mechanics, model its lift and drag characteristics on the computer. Once problems are identified, designs can immediately be modified and tested again on the computer, in the simulation environment.

The analogy to transportation engineering should be clear. If we want to test how well a new transportation facility will work, it is not practical to simply build it: it may be too expensive, it may affect real people or neighborhoods adversely, and it may disrupt existing transportation operations.

The advantages of being able to test a possible design for an intersection, or an arterial, or a freeway before we actually build it are apparent. We thus should be able to see the advantages of having a simulation model available to test the operation of a facility (and the alternative designs) before it is built and the consequences irreversible.

We can now define simulation more formally as it applies to transportation engineering[1]:

> "The purpose of a simulation experiment is to represent or model the actions and interactions of the elements of a system so that the effectiveness or level of service of the system can be determined for any set of design conditions. … Thus simulation can enable the engineer to predict the operations on a facility prior to its construction, and thus it can permit selection of the design components that will provide for the most effective flow."

There are several important "learning objectives" in this laboratory problem. The following list describes the major concepts that you will understand as a result of completing this problem:

1. What is simulation?
2. Why do we do simulation?
3. What are the elements of the simulation process?
4. What is interval-oriented vs. event-oriented simulation?
5. How do you use a computer spreadsheet to do a simulation experiment?

2. ELEMENTS OF A SIMULATION MODEL

A simulation model includes the following elements: (1) generation of random inputs, (2) processing of these inputs through the system, (3) monitoring the system performances (as defined by the appropriate measure of effectiveness, or MOE) during the test or simulation, and (4) keeping track of time using the system clock. The generation of inputs and the system clock are considered in this section.

Let's consider two examples of representing the model input, one with a discrete input and the other with a continuous input.

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[1] M. Wohl and B. Martin. **Traffic Systems Analysis.** McGraw-Hill Book Company, New York, 1965.

Discrete Variable Input Example

Consider vehicles approaching an intersection. Let's say that on the average, fifteen percent of the vehicles turn left, sixty percent proceed straight through, while twenty-five percent turn right. We can list each of these discrete events and its corresponding probability in Table 1.

Table 1. Probability Table for Discrete Events

Event	Probability
Left turn	0.15
Through	0.60
Right turn	0.25

Suppose that we had a method to generate random numbers between zero and one. Each time we pick a new random number, we then pick the turning movement of the vehicle at the intersection. In effect, we are "generating" an event: a vehicle first arriving at an intersection and then making a specific movement at the intersection. This is what we mean by "generation of random inputs." We can add a third column to the table above that lists cumulative probabilities that corresponds to the generation of random numbers between zero and one (See Table 2).

Table 2. Random Number Ranges for Discrete Events

Event	Probability	Range of Random Numbers
Left turn	0.15	0.00 - 0.15
Through	0.60	0.16 – 0.75
Right turn	0.25	0.76 – 1.00

Continuous Variable Input Example

Or suppose that we need to generate a vehicle headway, a continuous variable. The probability of a given gap (or headway) between two vehicles that is greater than *H* seconds is given by the negative exponential distribution, if vehicle arrivals are random.

$$P[h \geq H] = e^{-\lambda H}$$

where λ is the mean arrival rate.

Let's assume that we can generate a random number R_n between zero and one that represents the probability $P[h \geq H]$.

$$R_n = P\big[h \geq H\big] = e^{-\lambda H}$$

Solving for the headway H:

$$R_n = e^{-\lambda H}$$

$$\ln(R_n) = -\lambda H$$

$$H = -\frac{1}{\lambda}\ln(R_n)$$

Or, in words, you give me a random number R_n and I'll give you a headway H.

Simulation Clock

Related to the generation of random inputs (either discrete events or continuous times between events) is the method of keeping track of the simulation clock. *Interval-oriented* simulation updates the clock by a constant time interval, while *event-oriented* simulation updates the clock to the next event that triggers a change in the system state.

3. EXAMPLE: EVENT-ORIENTED SIMULATION MODEL

In this section we will consider the development of an event-oriented simulation model of a two-way stop-controlled (TWSC) intersection. We will first develop a "manual" version of the model. Once we see the steps involved in the simulation model, we can then use a spreadsheet to "automate" the simulation process.

Consider the operation of a TWSC intersection. Traffic on the major street arrives in a random manner and passes through the intersection unimpeded, without having to stop. Traffic on the minor street arrives (also in a random manner) at the stop sign, checks traffic on the major street, and crosses only when there is a gap in the major street traffic of sufficient size. How can we describe this operation in a form appropriate for a simulation model?

Case 1: Empty Server

If there are no vehicles waiting at the stop sign on the minor street when a new vehicle arrives, the following four steps represent the events of the simulation process.

Step 1: Generate an *arrival headway* H for the minor street vehicle based on the average flow rate for minor street traffic. The *arrival time* AT for the vehicle is the arrival time for the previous minor street vehicle plus the arrival headway.

$$AT_i = H_i + AT_{i-1}$$

Step 2: Compute the time that the minor street vehicle begins to wait for the available gap, or begins to wait for *service*. This time is also called the *service start time* (SST). Note that since no other vehicles are waiting at the stop line, the service start time is equal to the arrival time.

$$SST_i = AT_i$$

Step 3: Generate the *service time* ST for the minor street vehicle based on the gaps available in the minor street traffic. Determine the *service end time* SET, or the time that the vehicle leaves the server and enters the intersection.

$$SET_i = SST_i + ST_i$$

Table 3 lists and defines the variables for case 1. It is important to note that three of the variables represent the clock time that an event occurs while two of the variables represent the intervals between the occurrence of these events.

Table 3 Variables for Case 1 (Empty Server)

Variable	Definition	Type of Variable
AT_i	arrival time for vehicle i	Clock time
H_i	headway for vehicle i (the time between the arrival of vehicle i-1 and vehicle i)	Interval
SST_i	service start time for vehicle i (the time that vehicle i arrives at the stop line)	Clock time
SET_i	service end time for vehicle i (the time that vehicle i is served and leaves the stop line)	Clock time
ST_i	service time for vehicle i, or the time that vehicle i spends waiting at the stop line	Interval

Case 2: Occupied Server
Suppose now that there are other vehicles waiting at the stop line when a new minor street vehicle arrives. This condition can be handled with a slight modification in the logic described above.

Step 1: Generate an *arrival headway* H for the minor street vehicle based on the average flow rate for minor street traffic. The *arrival time* AT for the vehicle is the arrival time for the previous minor street vehicle plus the arrival headway.

$$AT_i = H_i + AT_{i-1}$$

Step 2: Compute the time that the minor street vehicle begins to wait for the available gap, or begins to wait for *service*. This time is also called the *service start time* (SST). If there is no other vehicle waiting at the stop line, the service start time is equal to the arrival time. If, however, there is a vehicle already waiting at the stop line, the service start time is equal to the *service end time* (SET) for the previous vehicle.

$$SST_i = \begin{cases} AT_i, if\ AT_i > SET_{i-1} \\ SET_{i-1}, if\ AT_i \leq SET_{i-1} \end{cases}$$

Step 3: Generate a *service time* ST for the minor street vehicle based on the gaps available in the major street traffic. Determine the *service end time* SET, or the time that the vehicle leaves the server and enters the intersection.

$$SET_i = SST_i + ST_i$$

Step 4: If there is still time remaining in the simulation period, return to step 1.

Arrival Headway
But how do we generate the arrival headway H_i and the ST_i for each vehicle i? We will consider each separately.

Suppose that the flow rate on the minor street is v vehicles per hour, or λ vehicles per second. The average headway H is then

$$H = \frac{1}{\lambda}$$

If traffic arrivals on the minor street can be considered to be a random process, we can use a negative exponential distribution to represent the headway between any two vehicles. Thus the probability that any headway h will be greater than or equal to H is

$$P[h \geq H] = e^{-\lambda H}$$

If R_n is a random number between zero and one, then

$$R_n = P[h \geq H] = e^{-\lambda H}$$

Solving for H:

$$H = -\frac{1}{\lambda}\ln(R_n)$$

Service Time
Now suppose that the flow rate on the major street is v vehicles per hour. If the minor street vehicles require a minimum headway, or critical gap, t_c, for the first vehicle to enter the traffic stream, and t_f (the follow-up time) for each successive minor stream vehicle to utilize the same gap, the capacity flow rate for the minor street is:

$$c = \frac{v_c e^{-vt_c/3600}}{1 - e^{-v_c t_f/3600}}$$

The mean service rate for the minor stream vehicles is:

$$\mu = \frac{c}{3600}$$

The mean service time, ST, is just the reciprocal of the mean service rate.

$$ST = \frac{1}{\mu}$$

Since the mean service time is randomly distributed, the probability that the service time for a given minor stream vehicle is greater than some value ST is

$$P[t \geq ST] = e^{-\mu ST}$$

If R_n is a random number between zero and one, then

$$R_n = e^{-\mu ST}$$

Solving for ST:

$$ST = -\frac{1}{\mu}\ln(R_n)$$

Spreadsheet Model

We can now use a spreadsheet to give some structure to the simulation model. Table 4 illustrates a simulation period covering five vehicles. In this example, vehicle 2 arrives when the server is empty and thus its service can begin immediately on arrival (that is, the $SST_2 = AT_2$, Figure 1). However, when vehicle 3 arrives, vehicle 2 is still being served, so $SST_3 = SET_2$ (Figure 2).

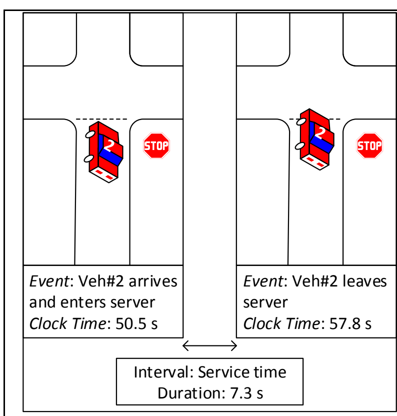

Figure 1 Case 1-Empty Server

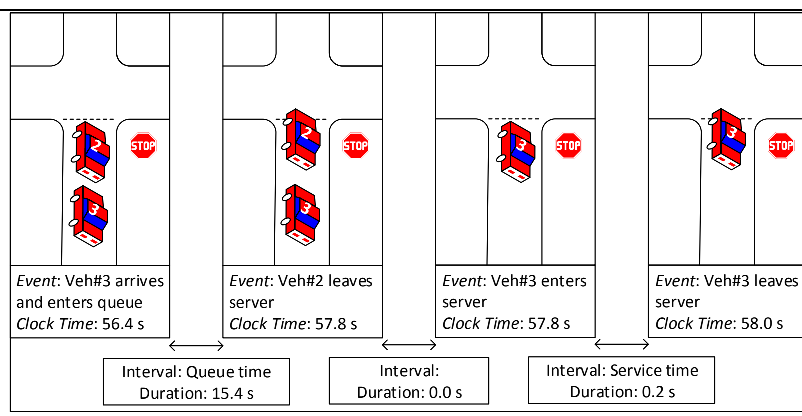

Figure 2 Case 2-Occupied Server

Table 4. Simulation Example in Spreadsheet Format

Veh#	Random number	H (interval)	AT (clock time)	Random number	SST (clock time)	ST (interval)	SET (clock time)
1	0.47233	13.5	0.0	0.03930	0.0	15.4	15.4
2	0.06045	50.5	50.5	0.21612	50.5	7.3	57.8
3	0.72227	5.9	56.4	0.96334	57.8	0.2	58.0
4	0.46346	13.8	70.2	0.05013	70.2	14.2	84.4
5	0.55474	10.6	80.8	0.27233	84.4	6.2	90.6